# Training PCFGs

Computational Linguistics

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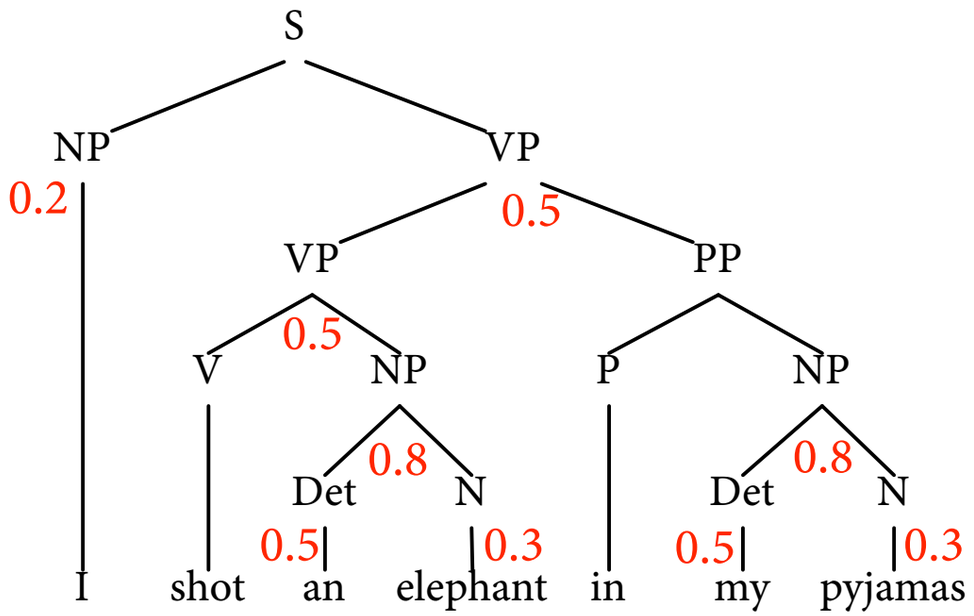
# Probabilistic CFGs

| | | | |
|---|---|---|---|
| S → NP VP | [1.0] | VP → V NP | [0.5] |
| NP → Det N | [0.8] | VP → VP PP | [0.5] |
| NP → i | [0.2] | V → shot | [1.0] |
| N → N PP | [0.4] | PP → P NP | [1.0] |
| N → elephant | [0.3] | P → in | [1.0] |
| N → pyjamas | [0.3] | Det → an | [0.5] |
| | | Det → my | [0.5] |

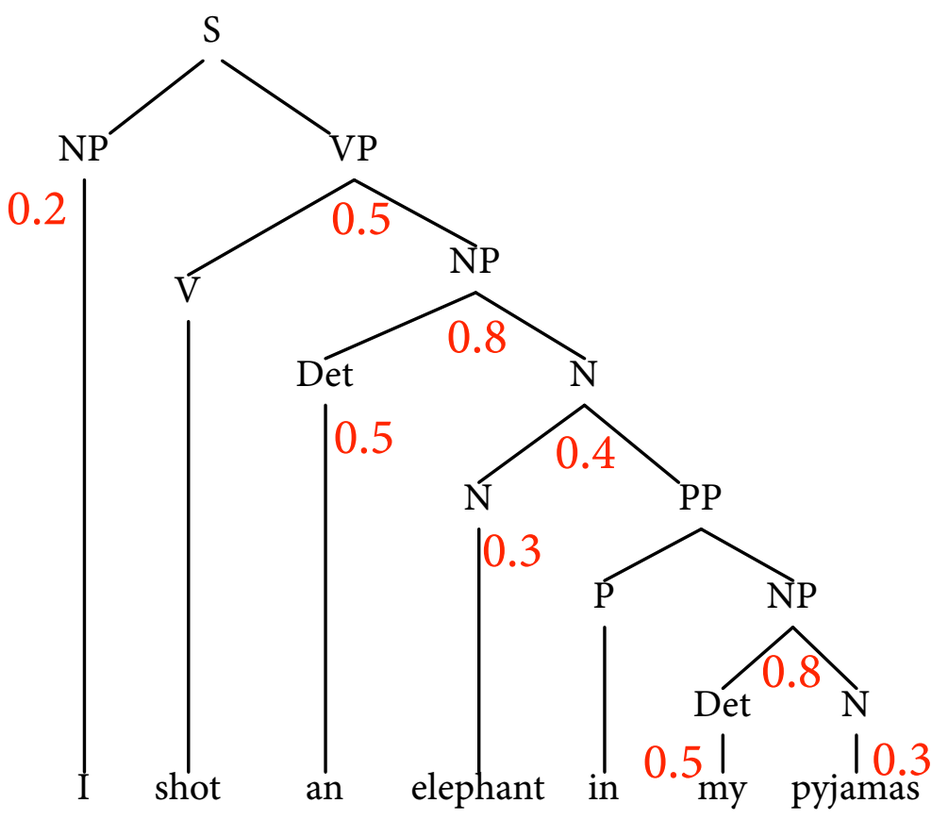
(let's pretend for simplicity that Det = PRP$)

# Parse trees

p = 0.00072




p = 0.00057




"correct" = more probable parse tree

# Today

- Parameters of PCFG = rule probabilities.

- How do we learn parameters from corpora?
  - maximum likelihood estimation
  - “hard EM” using Viterbi
  - “soft EM” using the inside-outside algorithm

# ML Estimation

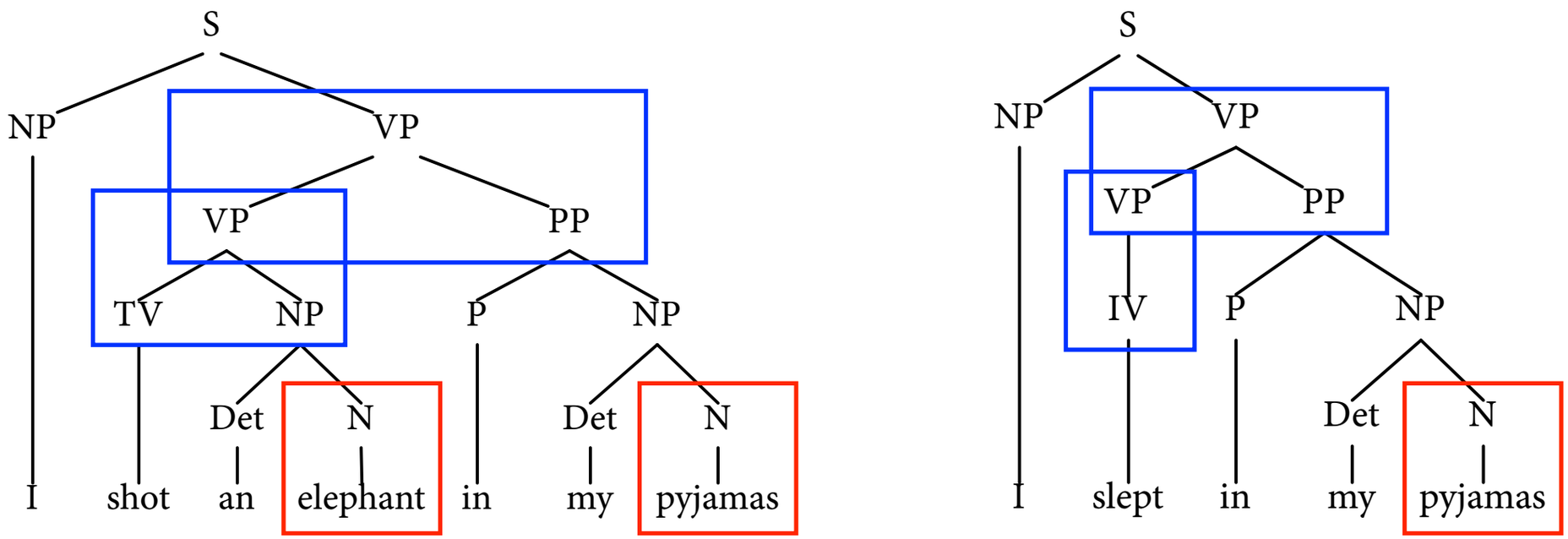
- Assume we have a treebank.
  - that is, every sentence annotated by hand with its "correct" parse tree

- Then we can use MLE to obtain rule probabilities:

$$P(A \to w) = \frac{C(A \to w)}{C(A \to \bullet)} = \frac{C(A \to w)}{\sum_{w'} C(A \to w')}$$

- Standard way of parameter estimation in practice. Works well, smoothing only needed for unknown words (or replace by POS tags).

# Example




| | | | |
|---|---|---|---|
| N → N PP | [0] | VP → TV NP | [1/4] |
| N → elephant | [1/3] | VP → IV | [1/4] |
| N → pyjamas | [2/3] | VP → VP PP | [1/2] |

# Unsupervised estimation

- MLE works okay *for English*.
  - German: Tiger treebank exists, but is hard for PCFGs, e.g. because of free word order.
  - most other languages: phrase structure annotations unavailable, expensive to create → unsupervised methods?

- Unsupervised methods:
  - provide CFG, learn parameters from unannotated corpus
  - show first “hard EM”, then “soft EM”
  - ideas instructive and generalize to other problems

# "Hard" aka Viterbi EM

- In the absence of syntactic annotations, learner must invent its own parse trees.

- Viterbi EM:
  - start with some parameter estimate
  - produce "syntactic annotations" by computing best tree for each sentence using Viterbi
  - apply MLE to re-estimate parameters
  - repeat as long as needed

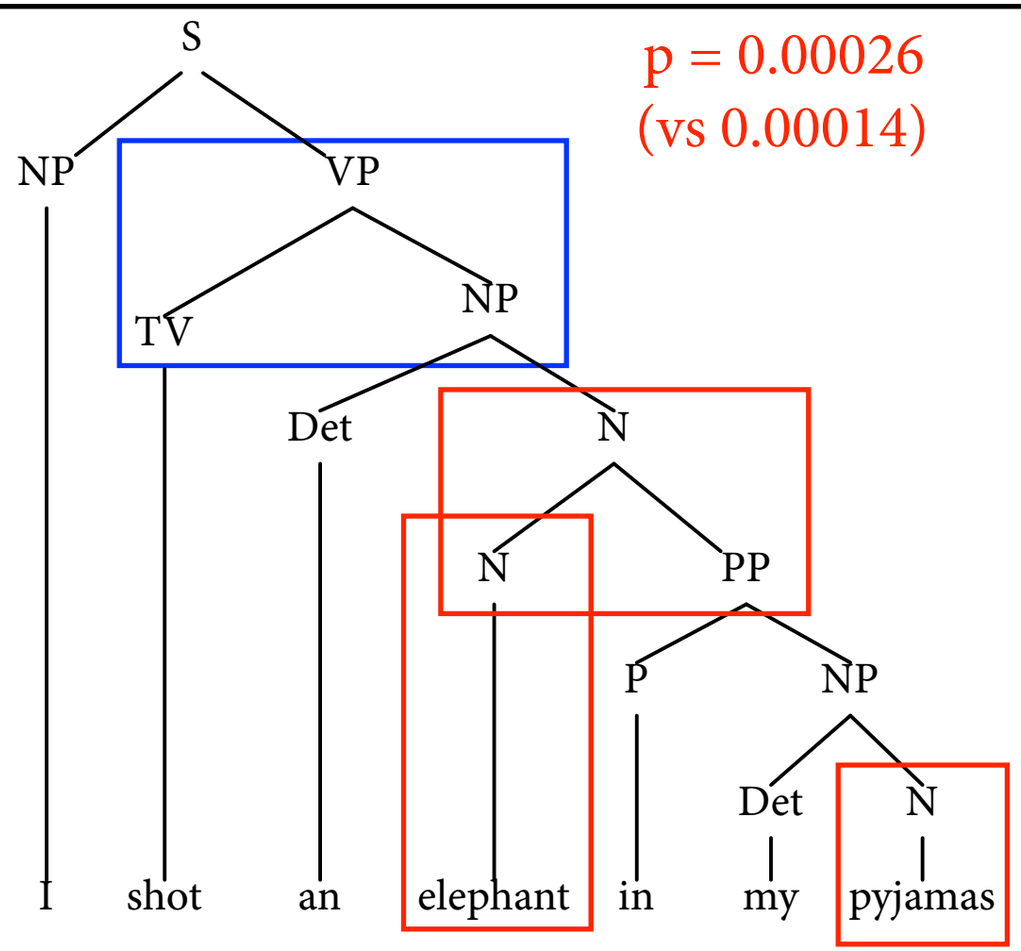
- This is *not* real EM!

# Example

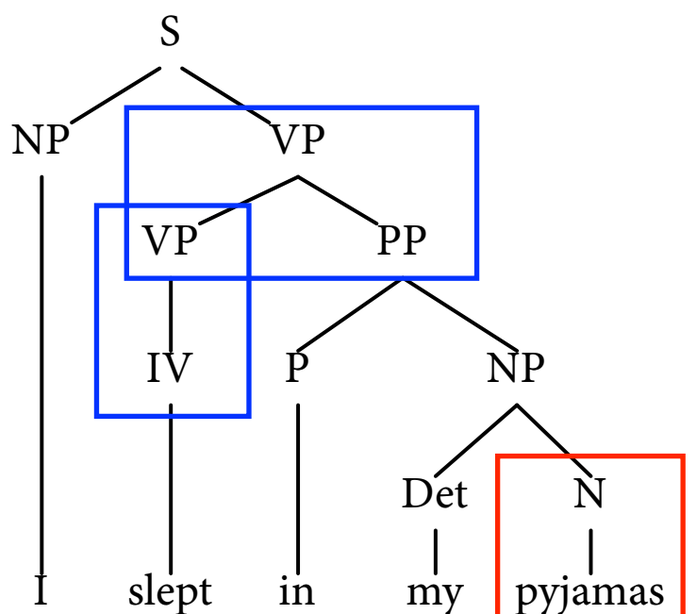
1

| | | | |
|---|---|---|---|
| N → N PP | [0.6] | VP → TV NP | [1/3] |
| N → elephant | [0.2] | VP → IV | [1/3] |
| N → pyjamas | [0.2] | VP → VP PP | [1/3] |

p = 0.00026
(vs 0.00014)




p = 0.00178

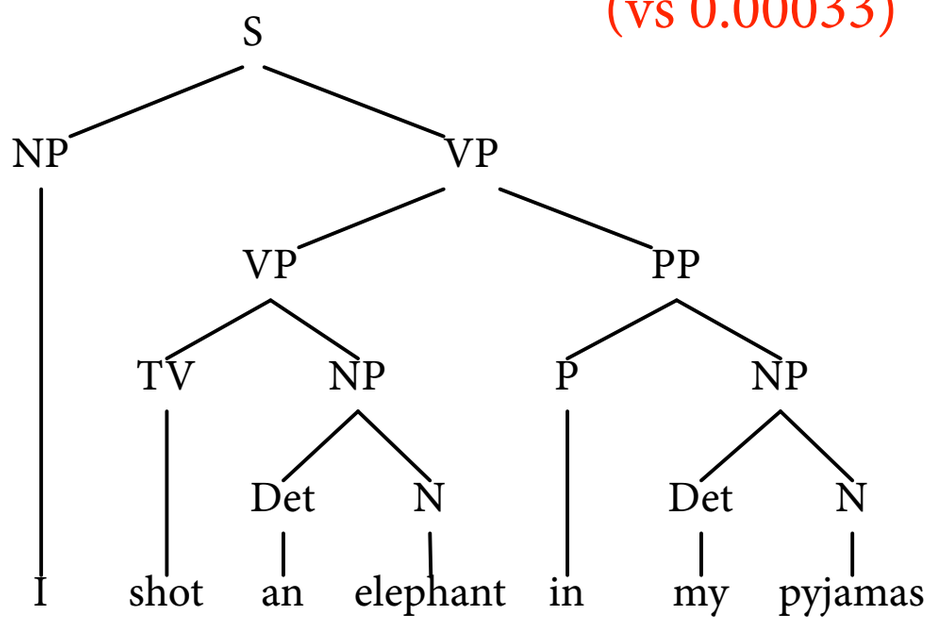


# MLE on Viterbi parses

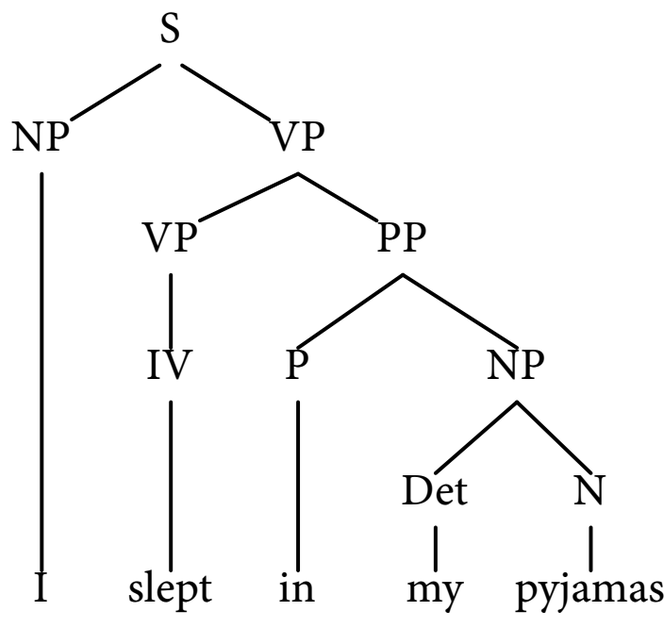
2

| | | | |
|---|---|---|---|
| N → N PP | [1/4] | VP → TV NP | [1/3] |
| N → elephant | [1/4] | VP → IV | [1/3] |
| N → pyjamas | [1/2] | VP → VP PP | [1/3] |

p = 0.00044
(vs 0.00033)




p = 0.00889



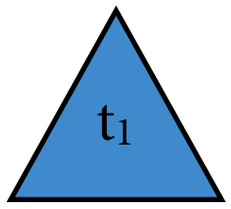
# Some things to note

- In this example, the likelihood increased.
  - this need not always be the case for Viterbi EM
- Viterbi EM commits to a single parse tree per sentence. This has advantages and disadvantages:
  - parse tree easy to compute, and can simply apply MLE
  - ignores all uncertainty we had about correct parse (winning parse tree takes all)

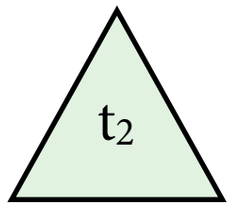
# Towards "real" (aka "soft") EM

idea: weighted counting of rules in all parse trees

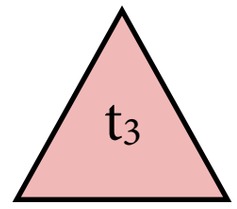
Viterbi-EM



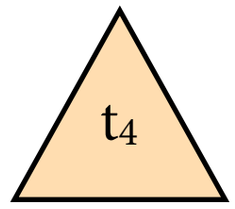

$1 \cdot C_{t1}(r)$




$+ \ 0 \cdot C_{t2}(r)$

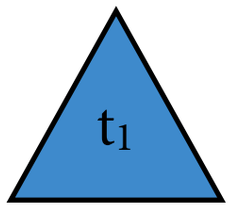


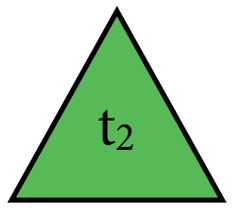

$+ \ 0 \cdot C_{t3}(r)$

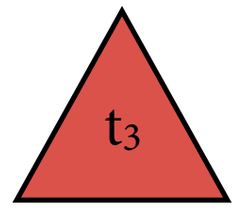


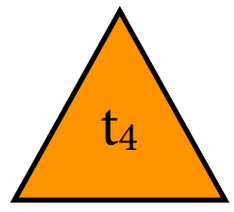

$+ \ 0 \cdot C_{t4}(r)$

EM




$$P(t_1 \mid w) \cdot C_{t1}(r) \ + \ P(t_2 \mid w) \cdot C_{t2}(r) \ + \ P(t_3 \mid w) \cdot C_{t3}(r) \ + \ P(t_4 \mid w) \cdot C_{t4}(r)$$

# Expected counts

- Define *expected count* of rule A → B C, based on previous parameter estimate.

$$E(A \rightarrow B\ C) = \sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_t(A \rightarrow B\ C)$$

- If we have them, can re-estimate parameters:

$$P(A \rightarrow B\ C) = \frac{E(A \rightarrow B\ C)}{\sum_r E(A \rightarrow r)}$$

- Challenge: How to compute E(A → B C) efficiently?
  - we assume grammars in CNF here

# Fundamental idea

$$E(A \to B\ C) = \sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_t(A \to B\ C)$$

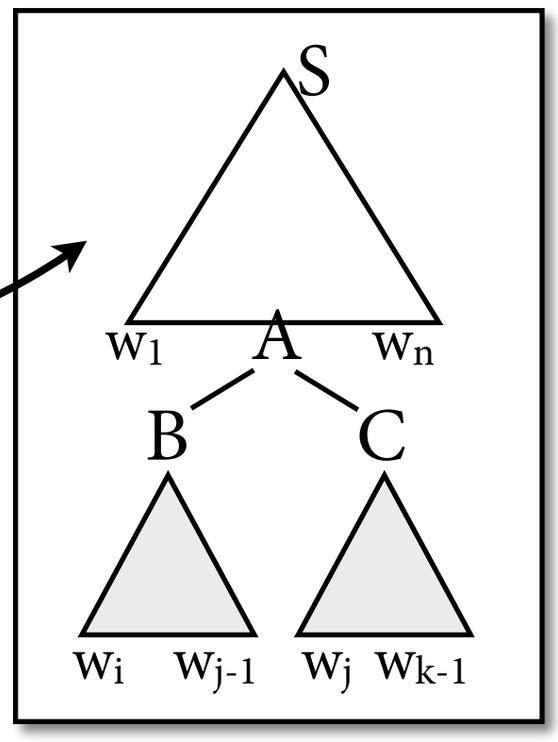
$$= \frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot C_t(A \to B\ C)$$

$$= \frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot \sum_{i,j,k} ||\text{rule for } i,j,k \text{ in } t \text{ is } A \to B\ C||$$

$$= \frac{1}{P(w)} \sum_{i,j,k} \left( \sum_{t \in \mathcal{T}} P(t) \cdot ||\text{rule for } i,j,k \text{ in } t \text{ is } A \to B\ C|| \right)$$

$$= \frac{1}{P(w)} \sum_{i,j,k} \left( \sum_{t \text{ of this form}} P(t) \right)$$

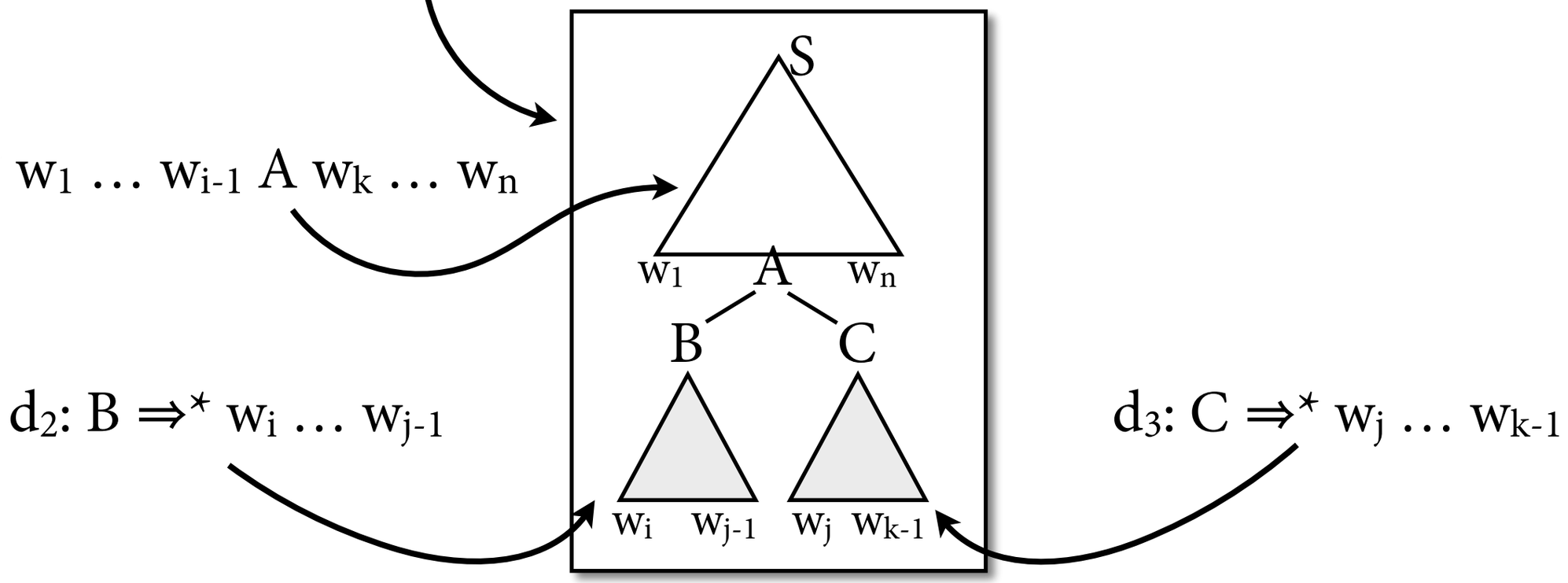
call this term μ(A → B C, i, j, k)




(note that P(t, w) = P(t))

# Computing μ

$$\mu(A \to B\ C, i, j, k) = \sum_{t \text{ of this form}} P(t)$$



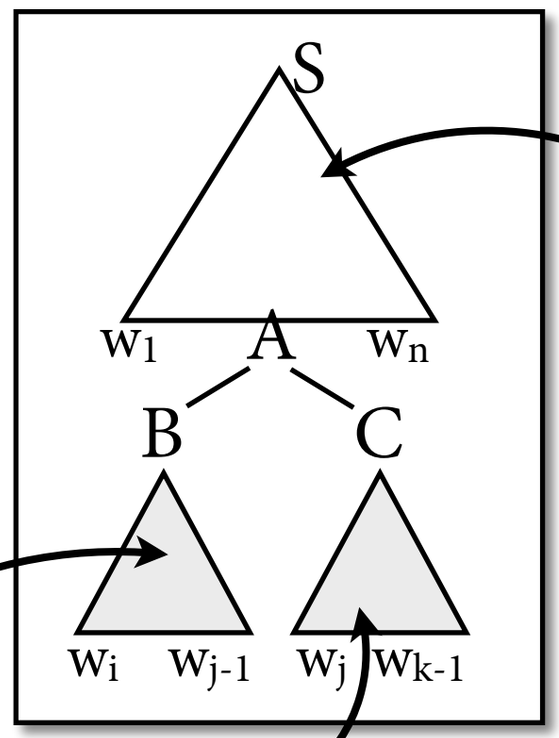

$$\mu(A \to B\ C, i, j, k) = \sum_{t \text{ of this form}} P(t)$$

$$= \sum_{t \text{ of this form}} P(d_1) \cdot P(A \to B\ C) \cdot P(d_2) \cdot P(d_3)$$

$$= \left(\sum_{d_1} P(d_1)\right) \cdot P(A \to B\ C) \cdot \left(\sum_{d_2} P(d_2)\right) \cdot \left(\sum_{d_3} P(d_3)\right)$$

# Computing μ

$$\mu(A \rightarrow B\ C, i, j, k) = \sum_{t \text{ of this form}} P(t) = \alpha(A, i, k) \cdot P(A \rightarrow B\ C) \cdot \beta(B, i, j) \cdot \beta(C, j, k)$$




*inside probability*

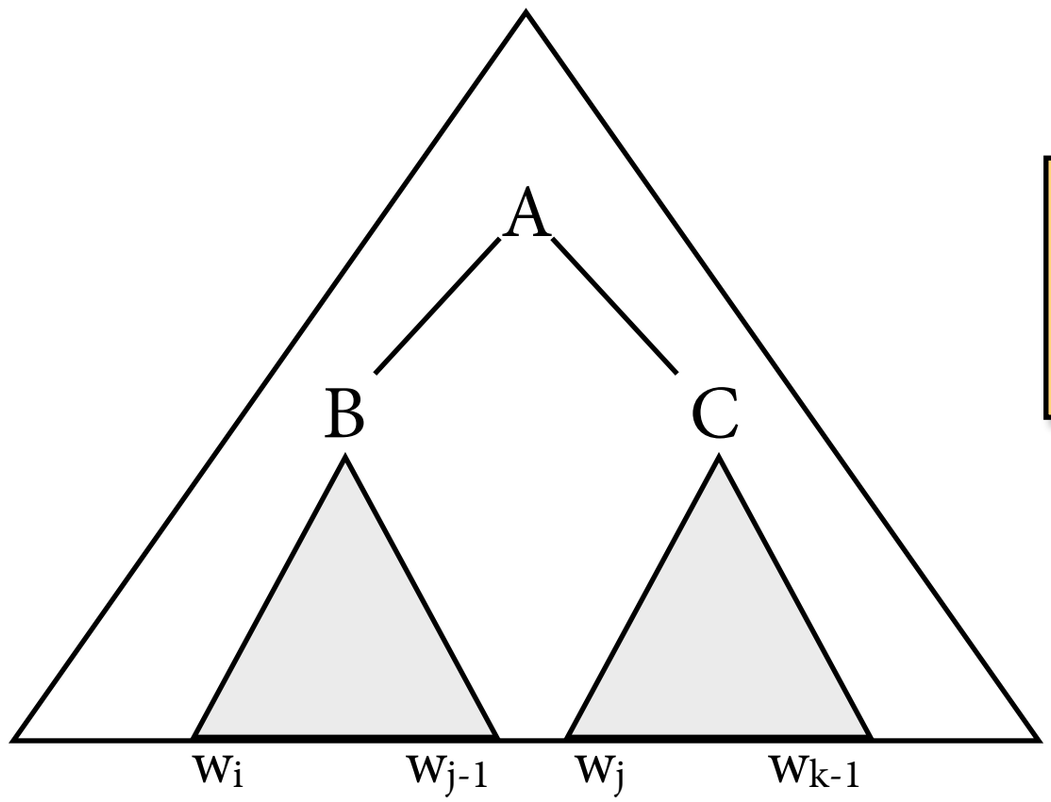
$$\beta(B, i, j) = \sum_{B \overset{d}{\Rightarrow}^{*} w_i \ldots w_{j-1}} P(d)$$

*outside probability*

$$\alpha(A, i, k) = \sum_{S \overset{d}{\Rightarrow}^{*} w_1 \ldots w_{i-1} A w_k \ldots w_n} P(d)$$

# Inside probabilities

$$\beta(B, i, j) = \sum_{B \overset{d}{\Rightarrow}^{*} w_i \ldots w_{j-1}} P(d)$$




special case:
P(w) = β(S,1,n+1)

$$\beta(A, i, i+1) = P(A \rightarrow w_i)$$

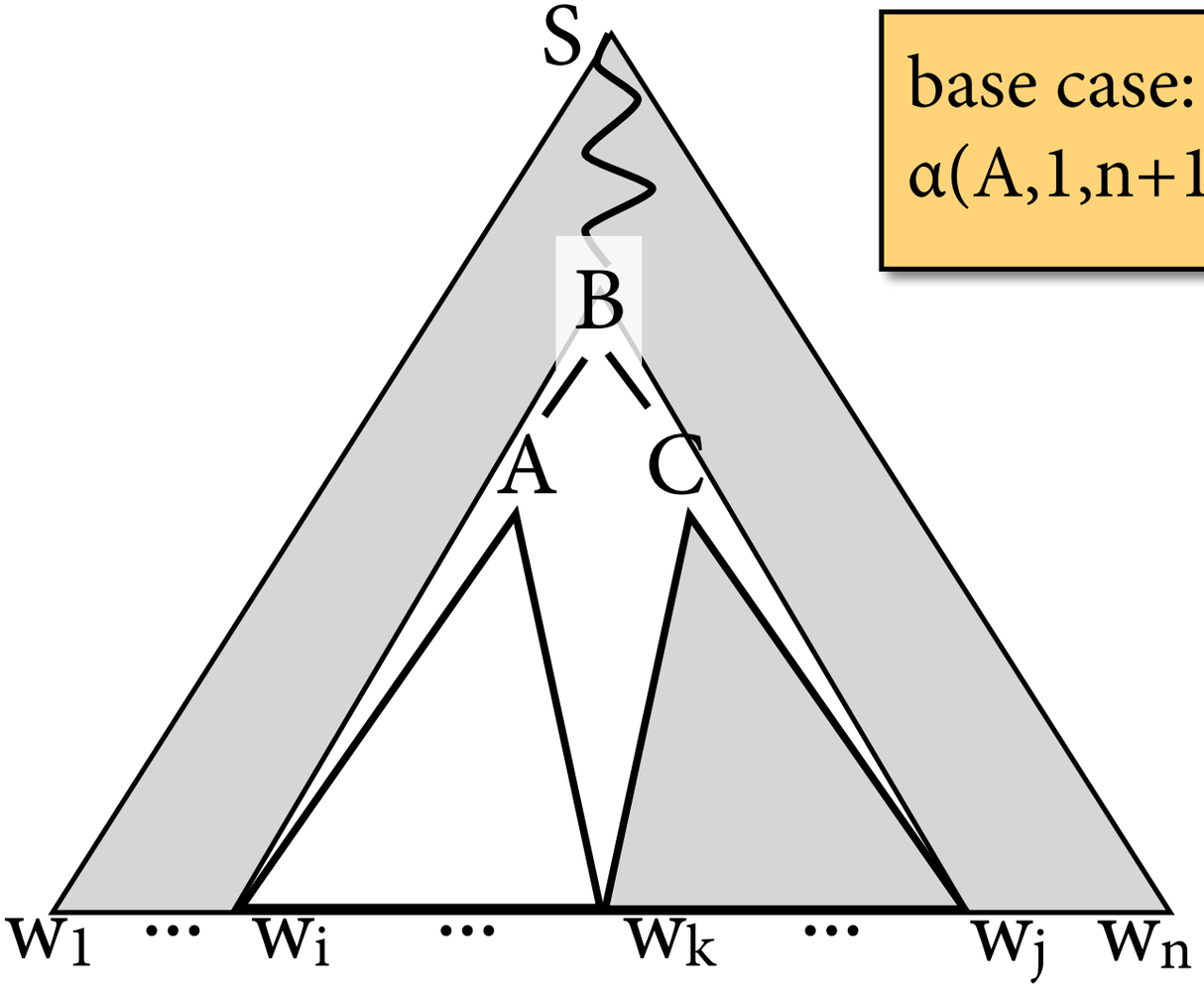
$$\beta(A, i, k) = \sum_{\substack{A \rightarrow B\ C \\ i<j<k}} P(A \rightarrow B\ C) \cdot \beta(B, i, j) \cdot \beta(C, j, k)$$

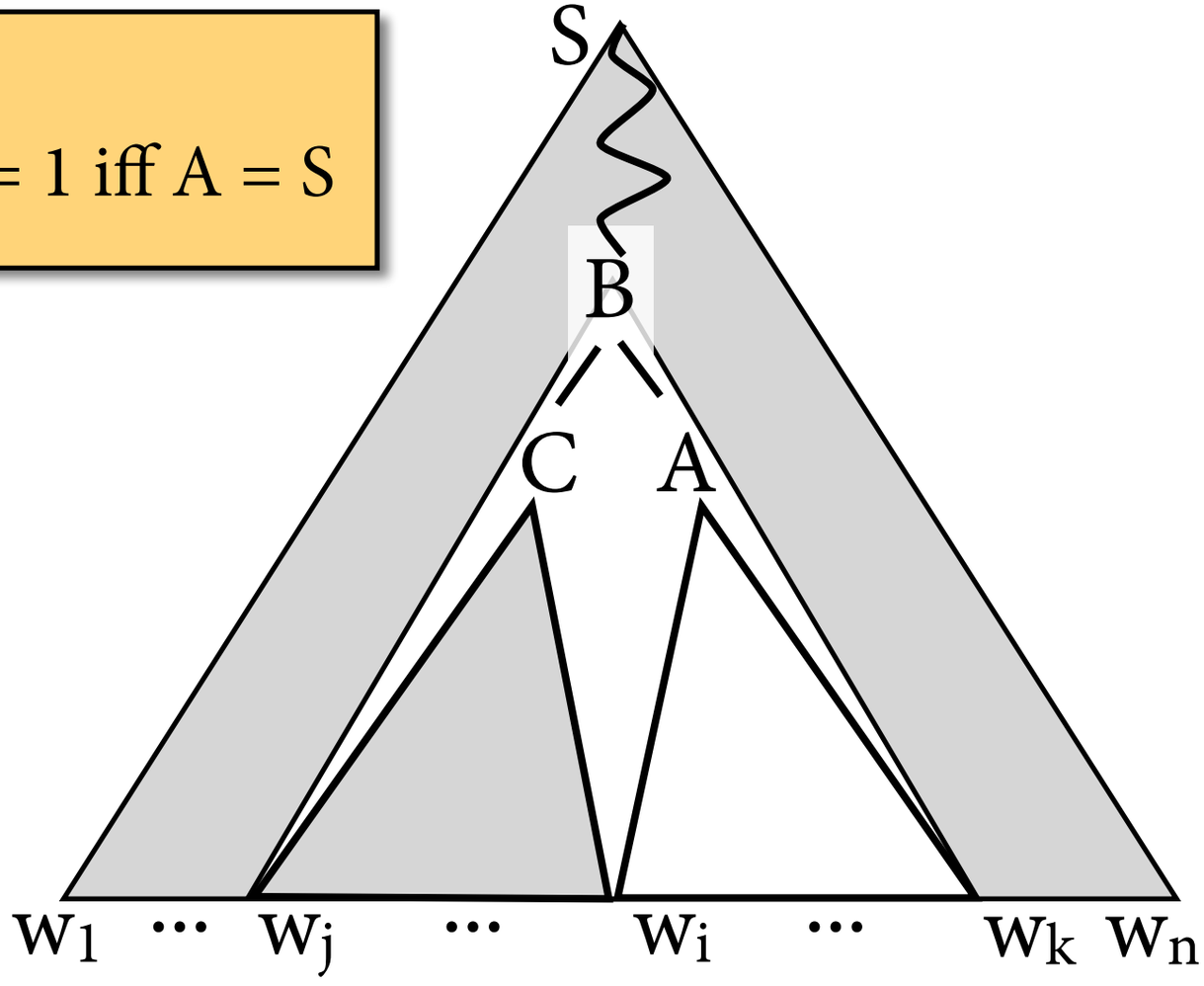
# Outside probabilities

$$\alpha(A,i,k) = \sum_{S \overset{d}{\Rightarrow}^{*} w_1 \ldots w_{i-1} A w_k \ldots w_n} P(d)$$

$$= \sum_{\substack{B \to A\ C \\ k<j\leq n}} P(B \to A\ C) \cdot \beta(C,k,j) \cdot \alpha(B,i,j) + \sum_{\substack{B \to C\ A \\ 1\leq j<i}} P(B \to C\ A) \cdot \beta(C,j,i) \cdot \alpha(B,j,k)$$

base case:
α(A,1,n+1) = 1 iff A = S

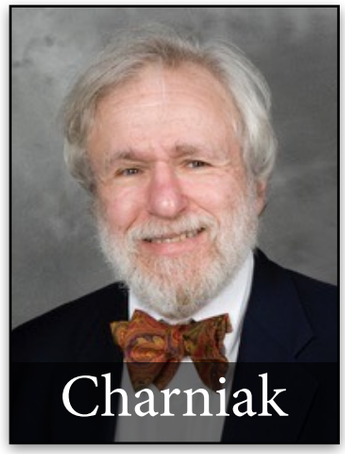


# The Inside-Outside Algorithm

- Start with some initial estimate of parameters.
- For each sentence w, compute $\alpha$, $\beta$, and $\mu$.
- Compute expected counts $E(A \rightarrow B\ C)$.
  - sum expected counts over all sentences
  - remember that $P(w) = \beta(S, 1, n+1)$
- Re-estimate $P(A \rightarrow B\ C)$ from expected counts.
- Iterate until convergence.

# Some remarks


Charniak


Pereira

- Inside-outside increases likelihood in each step.

- But huge problems with local maxima.
  - Carroll & Charniak 92 find 300 different local maxima for 300 different initial parameter estimates.
  - Improve by partially bracketing strings (Pereira & Schabes 92).

- Therefore, EM doesn't really work for totally unsupervised PCFG training.

- But extremely useful in refining existing grammars (Berkeley parser; see next time).

# Summary

- Learning parameters of PCFGs:
  - maximum likelihood estimation from raw text
  - “hard EM”: iterate MLE on Viterbi parses
  - EM: use inside-outside algorithm with expected rule counts

- PCFG parsing with MLE parse gets f-score in low 70’s. Will improve on this next time (state of the art: 93).

- Have assumed that CFG is given and only parameters are to be learned. Will fix this later in this course.