Probabilistic Context-free Grammars

Computational Linguistics

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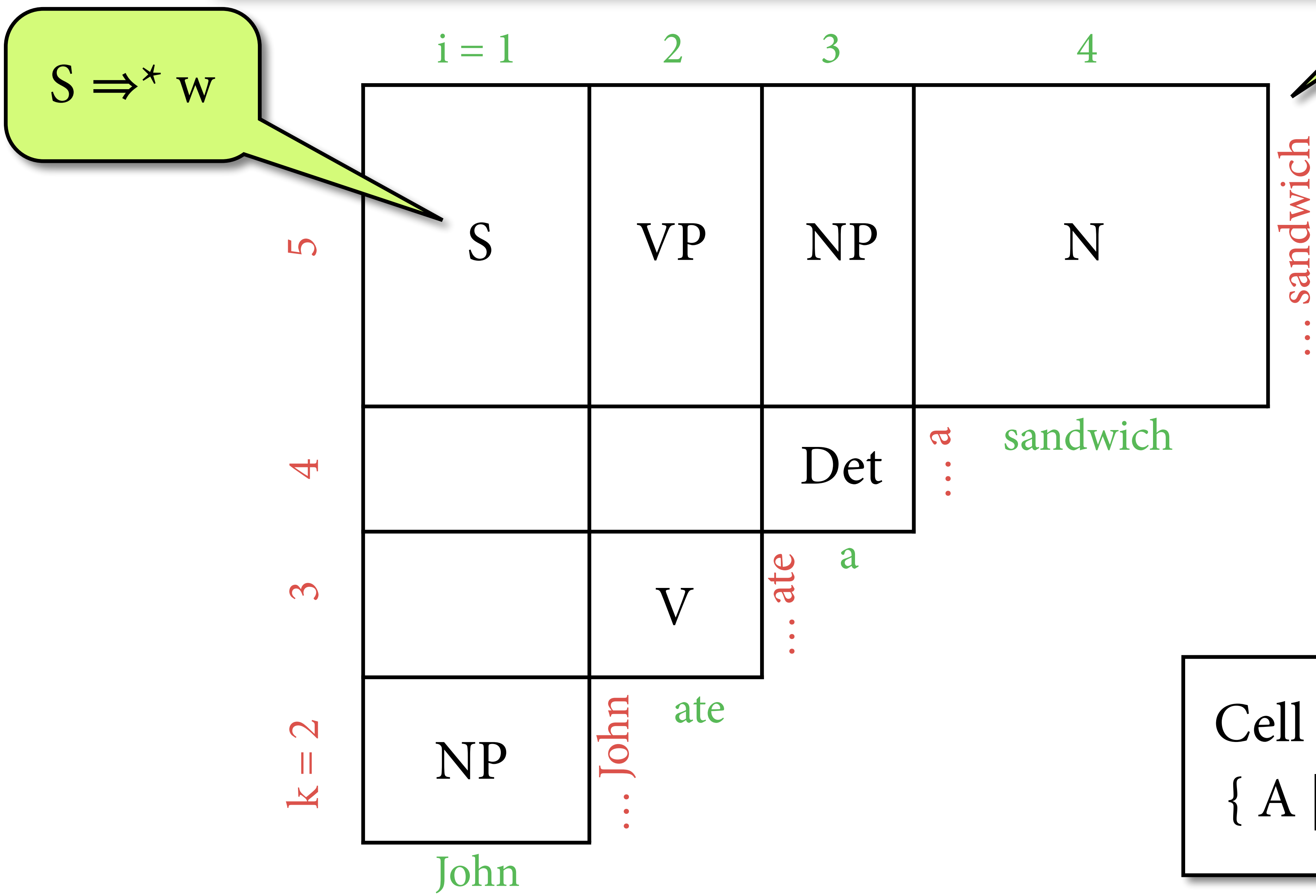
The CKY Recognizer

S → NP VP	V → ate	Det → a
NP → Det N	NP → John	N → sandwich
VP → V NP		

Chart



Cell at column i, row k:

$\{ A \mid A \Rightarrow^* w_i \ldots w_{k-1} \}$

CKY recognizer: pseudocode

Data structure: $Ch(i,k)$ eventually contains $\{A \mid A \Rightarrow^* w_i \dots w_{k-1}\}$ (initially all empty).

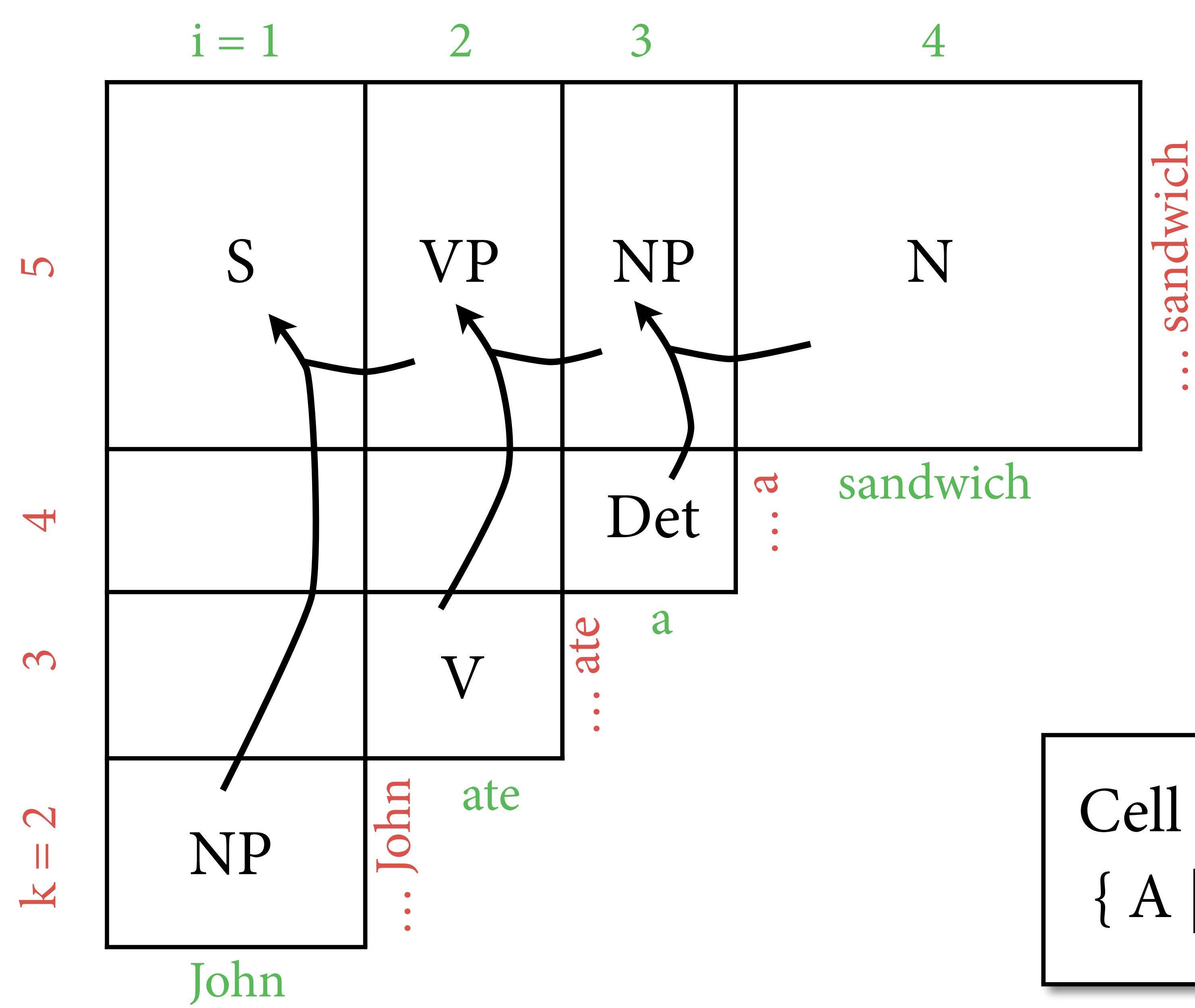
for each i from 1 to n:
    for each production rule $A \rightarrow w_i$:
        add A to Ch(i, i+1)

for each *width* b from 2 to n:
    for each *start position* i from 1 to n-b+1:
        for each *left width* k from 1 to b-1:
            for each $B \in Ch(i, i+k)$ and $C \in Ch(i+k,i+b)$:
                for each production rule $A \rightarrow B\ C$:
                    add A to Ch(i,i+b)

claim that $w \in L(G)$ iff $S \in Ch(1,n+1)$

Recognizer to Parser

S → NP VP	V → ate	Det → a
NP → Det N	NP → John	N → sandwich
VP → V NP		



Cell at column i, row k:
$\{ A \mid A \Rightarrow^* w_i \dots w_{k-1} \}$

Recognizer to Parser

- Parser: need to construct parse trees from chart.

- Do this by memorizing how each A ∈ Ch(i,k) can be constructed from smaller parts.
  - built from B ∈ Ch(i,j) and C ∈ Ch(j,k) using A → B C: store (B,C,j) in *backpointer* for A in Ch(i,k).
  - analogous to backpointers in HMMs

- Once chart has been filled, enumerate trees recursively by following backpointers, starting at S ∈ Ch(1,n+1).

Let’s play a game

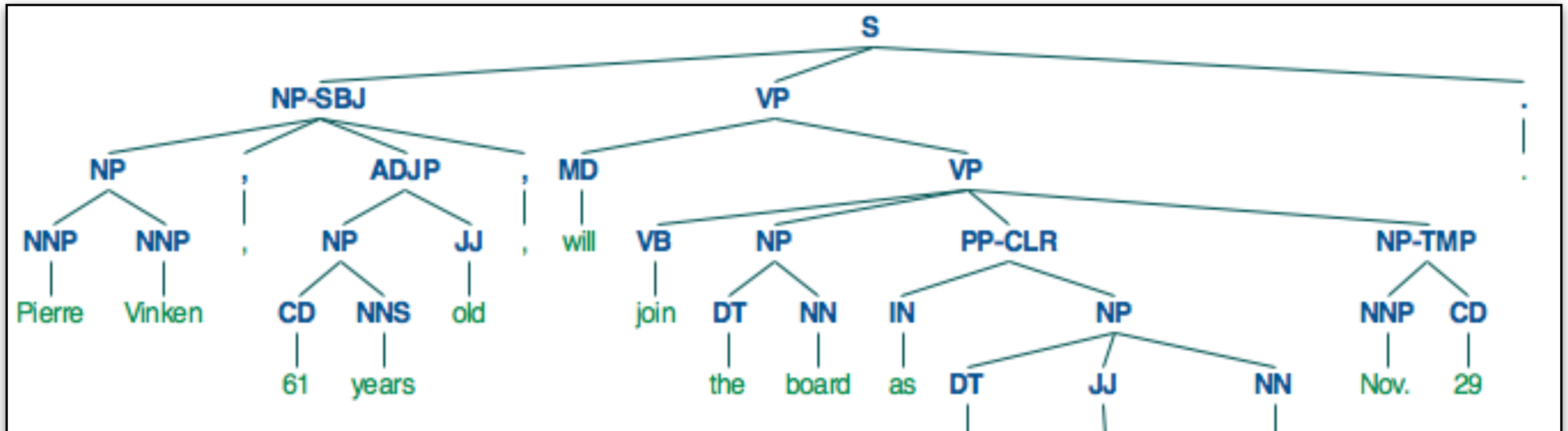
- Given a nonterminal symbol, expand it.

- You can take one of two moves:
  - expand nonterminal into a sequence of other nontermianls
  - use nonterminals S, NP, VP, PP, … or POS tags
  - expand nonterminal into a word

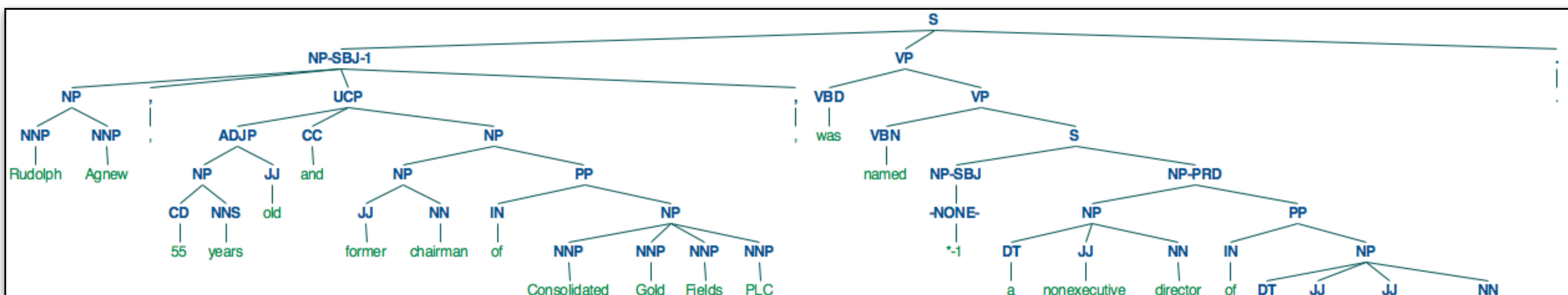
Penn Treebank POS tags

Tag	Description	Example	Tag	Description	Example
CC	Coordin. Conjunction	*and, but, or*	SYM	Symbol	+,%, &
CD	Cardinal number	*one, two, three*	TO	"to"	*to*
DT	Determiner	*a, the*	UH	Interjection	*ah, oops*
EX	Existential 'there'	*there*	VB	Verb, base form	*eat*
FW	Foreign word	*mea culpa*	VBD	Verb, past tense	*ate*
IN	Preposition/sub-conj	*of, in, by*	VBG	Verb, gerund	*eating*
JJ	Adjective	*yellow*	VBN	Verb, past participle	*eaten*
JJR	Adj., comparative	*bigger*	VBP	Verb, non-3sg pres	*eat*
JJS	Adj., superlative	*wildest*	VBZ	Verb, 3sg pres	*eats*
LS	List item marker	*1, 2, One*	WDT	Wh-determiner	*which, that*
MD	Modal	*can, should*	WP	Wh-pronoun	*what, who*
NN	Noun, sing. or mass	*llama*	WP$	Possessive wh-	*whose*
NNS	Noun, plural	*llamas*	WRB	Wh-adverb	*how, where*
NNP	Proper noun, singular	*IBM*	$	Dollar sign	$
NNPS	Proper noun, plural	*Carolinas*	#	Pound sign	#
PDT	Predeterminer	*all, both*	"	Left quote	(' or ")
POS	Possessive ending	*'s*	"	Right quote	(' or ")
PP	Personal pronoun	*I, you, he*	(	Left parenthesis	([, (, {, <)
PP$	Possessive pronoun	*your, one's*	)	Right parenthesis	( ],), }, >)
RB	Adverb	*quickly, never*	,	Comma	,
RBR	Adverb, comparative	*faster*	.	Sentence-final punc	(. ! ?)
RBS	Adverb, superlative	*fastest*	:	Mid-sentence punc	(: ; ... – -)
RP	Particle	*up, off*			

Some real trees



Penn Treebank, #0001

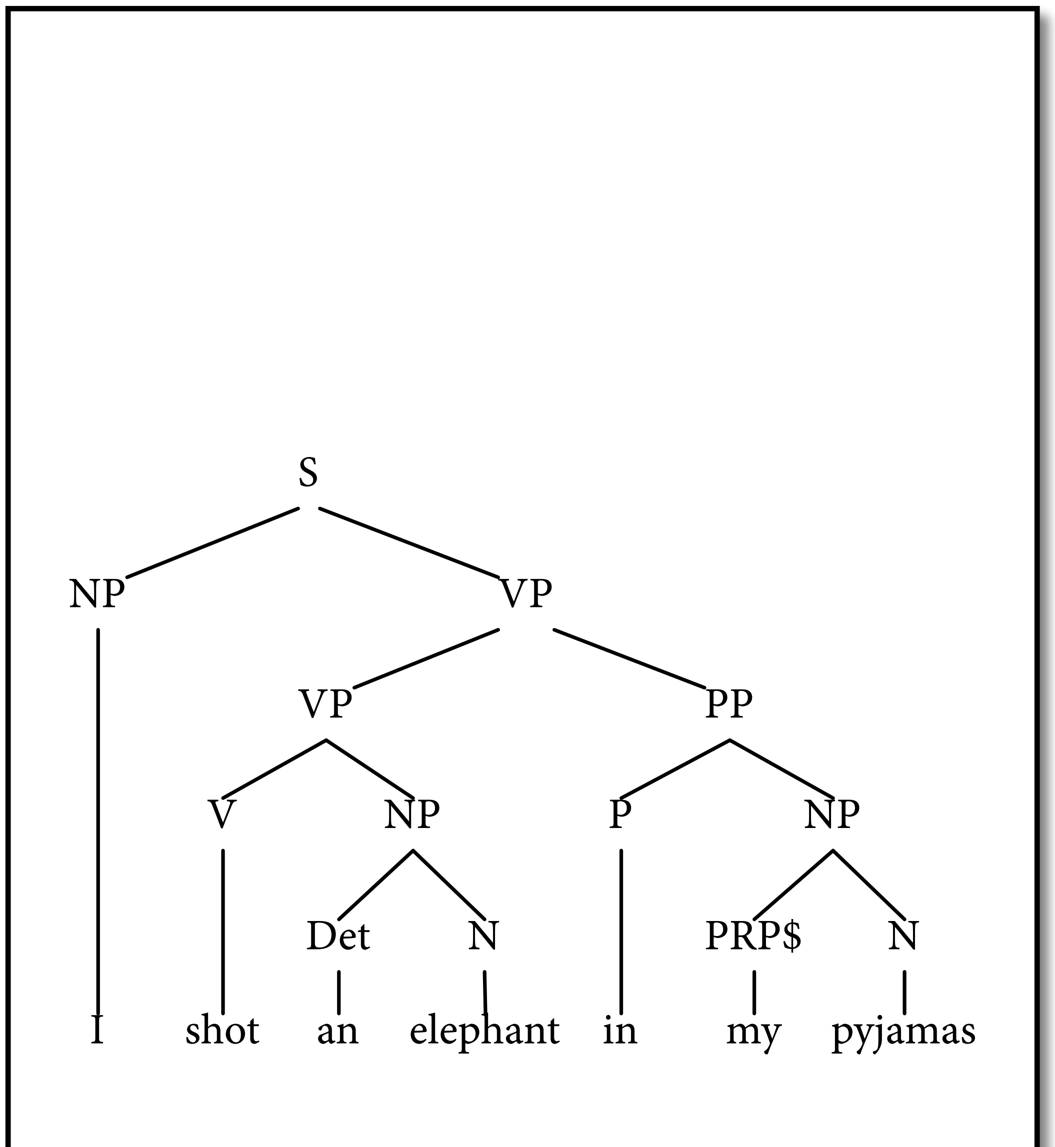


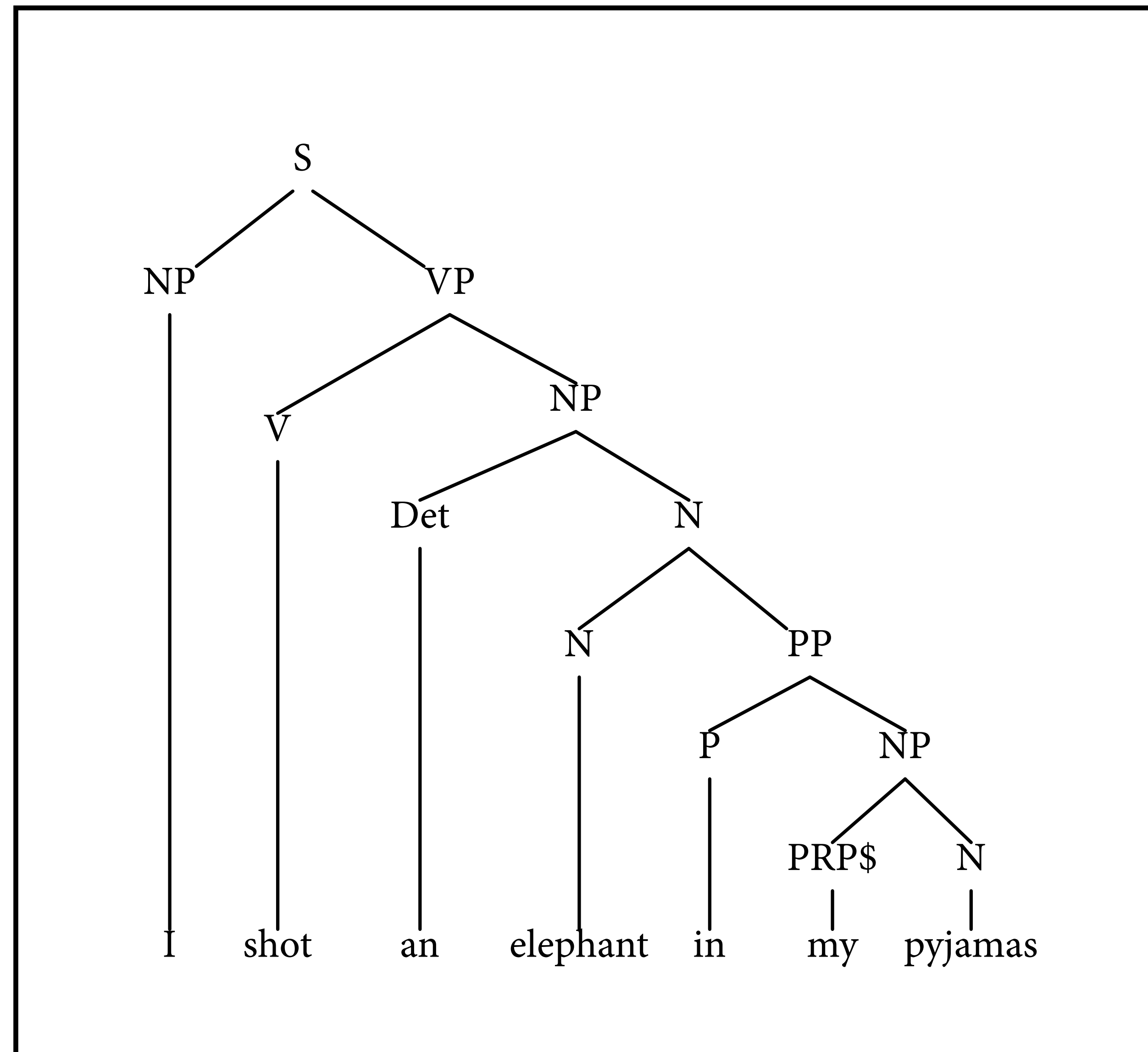
Penn Treebank, #0002

```
nltk.corpus.treebank.parsed_sents("wsj_0001.mrg")[0].draw()
```

Ambiguity

Need to *disambiguate:* find "correct" parse tree for ambiguous sentence.





How do we identify the "correct" tree?
How do we compute it efficiently? (Remember: exponential number of readings.)

Probabilistic CFGs

- A *probabilistic context-free grammar (PCFG)* is a context-free grammar in which
  - each production rule A → w has a probability P(A → w | A): when we expand A, how likely is it that we choose A → w?
  - for each nonterminal A, probabilities must sum to one:

    $$\sum_{w} P(A \to w \mid A) = 1$$

  - we will write P(A → w) instead of P(A → w | A) for short

An example

S → NP VP	[1.0]	VP → V NP	[0.5]
NP → Det N	[0.8]	VP → VP PP	[0.5]
NP → i	[0.2]	V → shot	[1.0]
N → N PP	[0.4]	PP → P NP	[1.0]
N → elephant	[0.3]	P → in	[1.0]
N → pyjamas	[0.3]	Det → an	[0.5]
		Det → my	[0.5]

(let's pretend for simplicity that Det = PRP$)

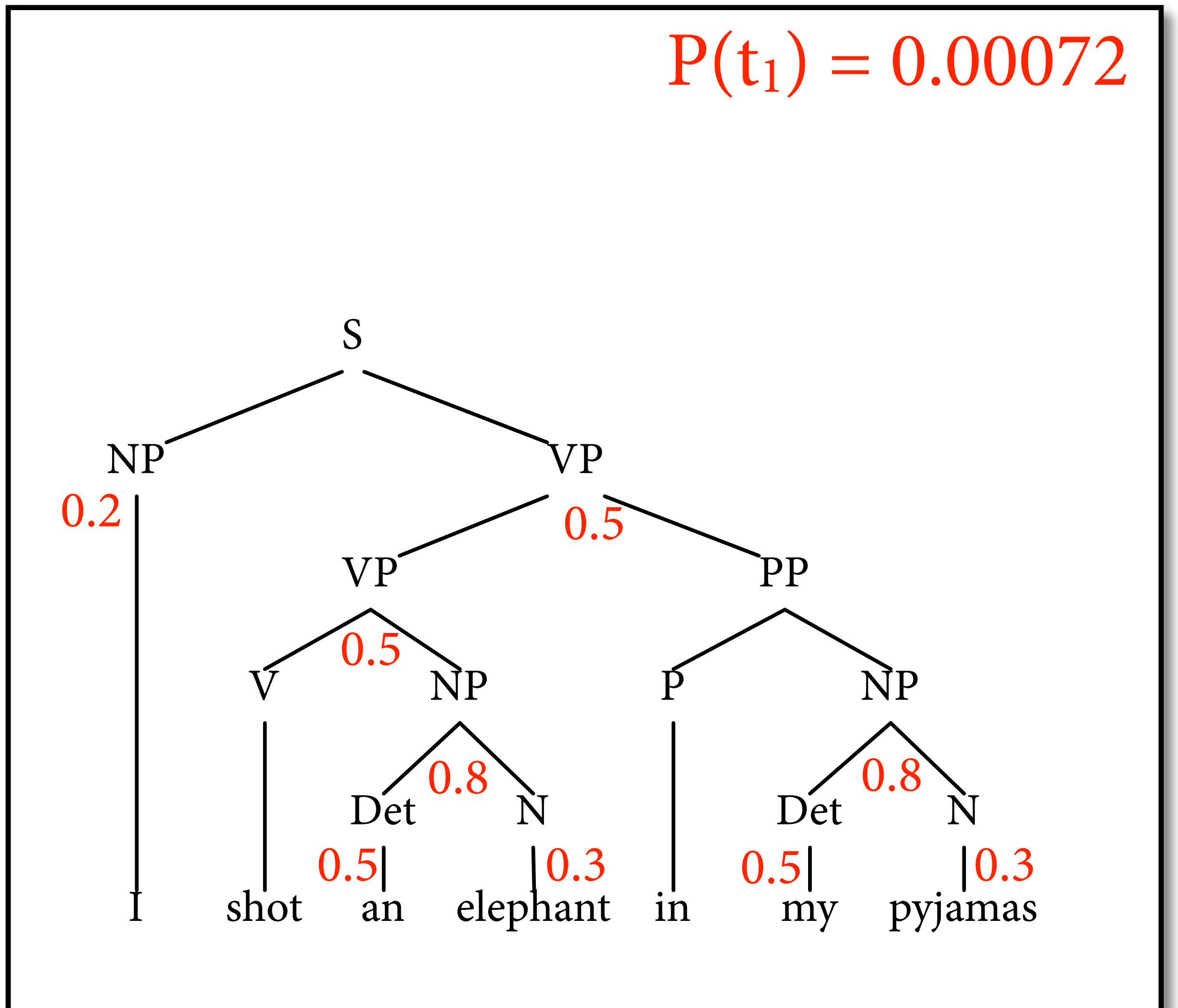
Generative process

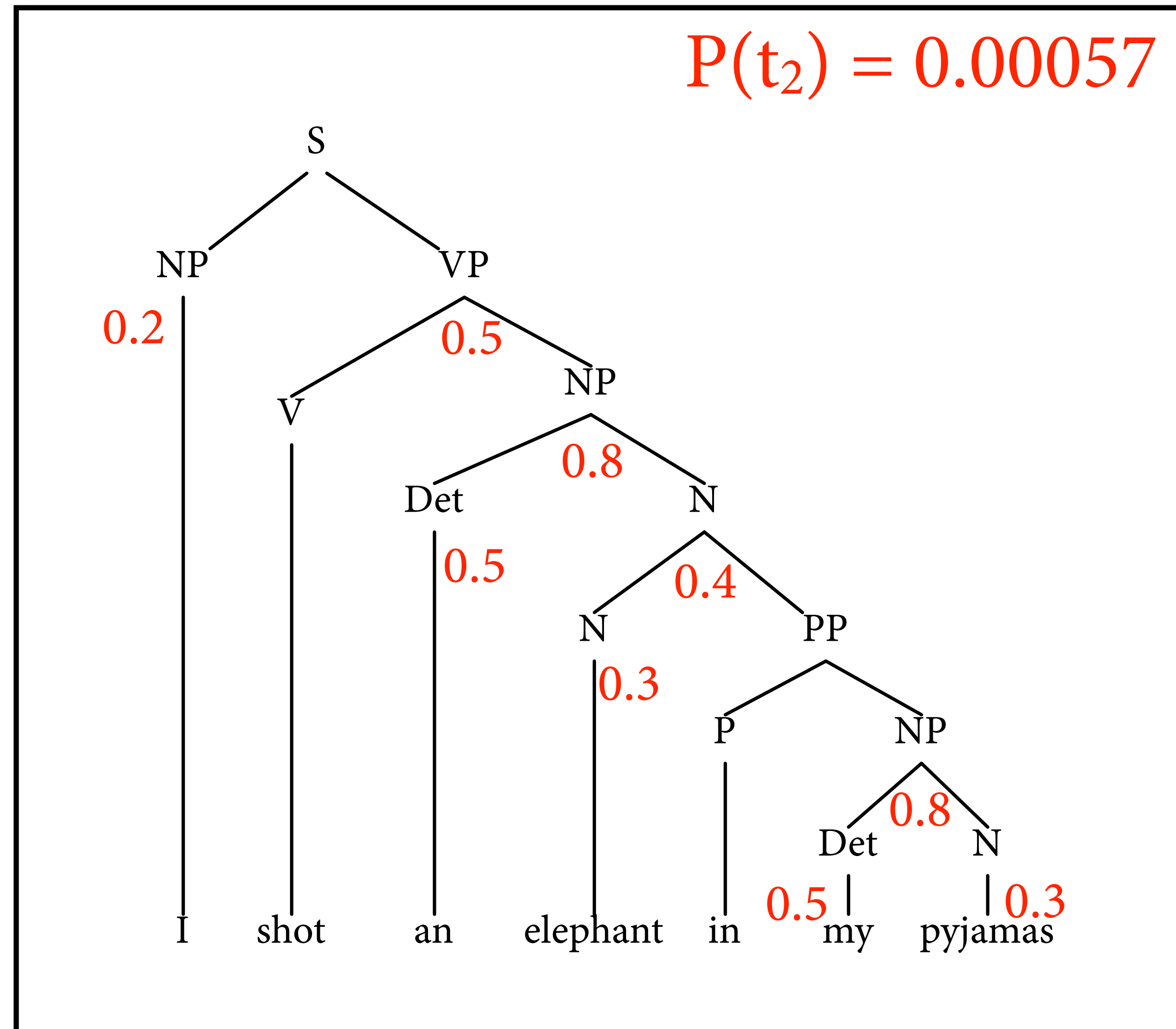
- PCFG generates random derivations of CFG.
  - each event (expand nonterminal by production rule) statistically independent of all the others

$$S \overset{1.0}{\Rightarrow} NP\ \ VP \overset{0.2}{\Rightarrow} i\ VP \overset{0.5}{\Rightarrow} i\ VP\ PP \Rightarrow^{*} i\ shot\ an\ elephant\ in\ my\ pyjamas \quad 0.00072$$

$$S \overset{1.0}{\Rightarrow} NP\ VP \overset{0.2}{\Rightarrow} i\ VP \overset{0.4}{\Rightarrow^{*}} i\ V\ Det\ N \underset{0.4}{\Rightarrow} i\ V\ Det\ N\ PP \Rightarrow^{*} i\ shot\ \ldots\ pyjamas \quad 0.00057$$

Parse trees





"correct" = more probable parse tree

Language modeling

- As with other generative models (e.g. HMMs), can define probability P(w) of string by marginalizing over its possible parses:

$$P(w) = \sum_{t \in \mathsf{parses}(w)} P(t)$$

- Can compute this efficiently with *inside probabilities*, see next time.

Disambiguation

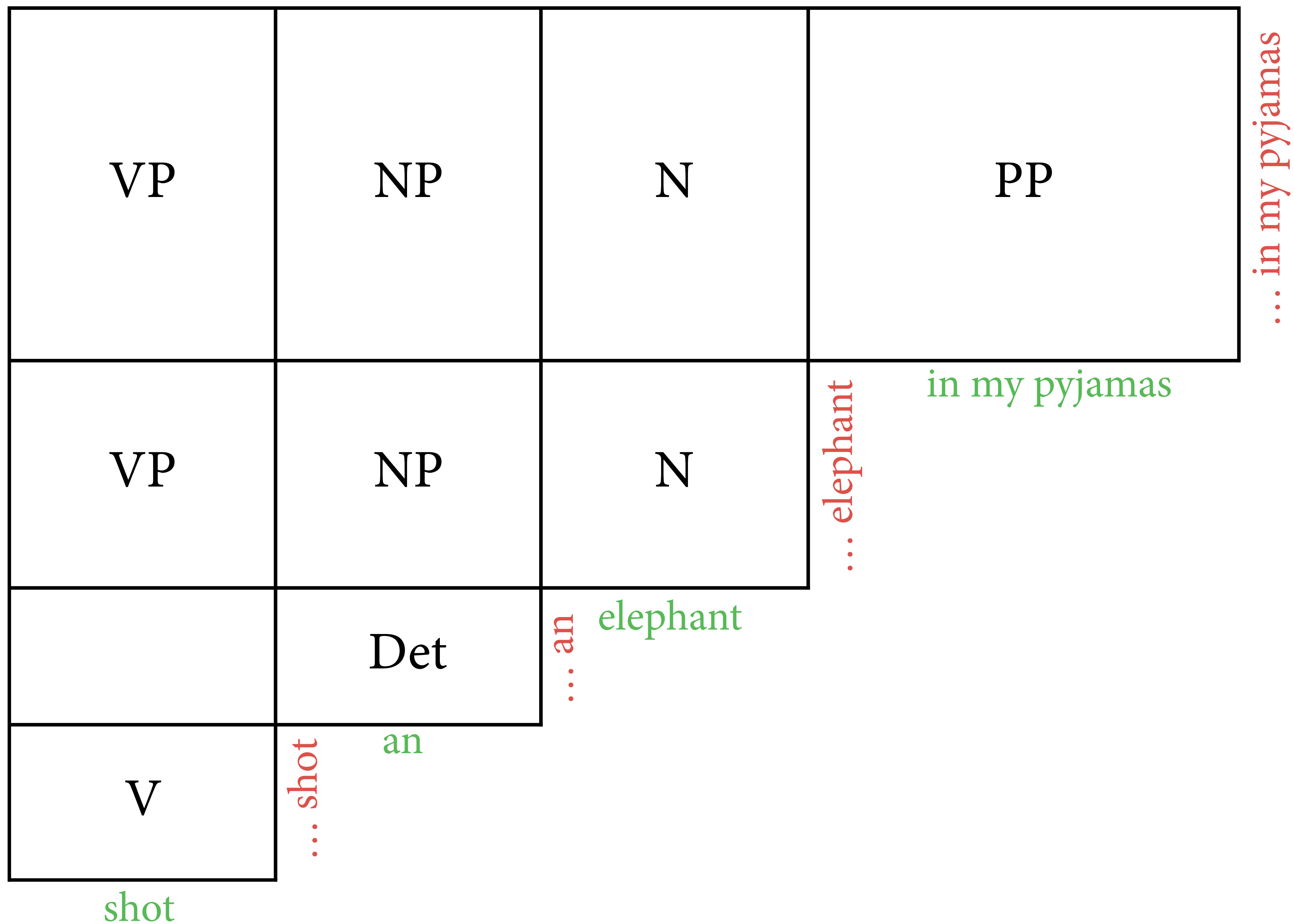
- Assumption: "correct" parse tree = the parse tree that had highest prob of being generated by random process, i.e. $\underset{t \in \mathsf{parses}(\mathsf{w})}{\operatorname{argmax}} \; P(t)$

- We use a variant of the Viterbi algorithm to compute it.

- Here, Viterbi based on CKY; can do it with other parsing algorithms too.

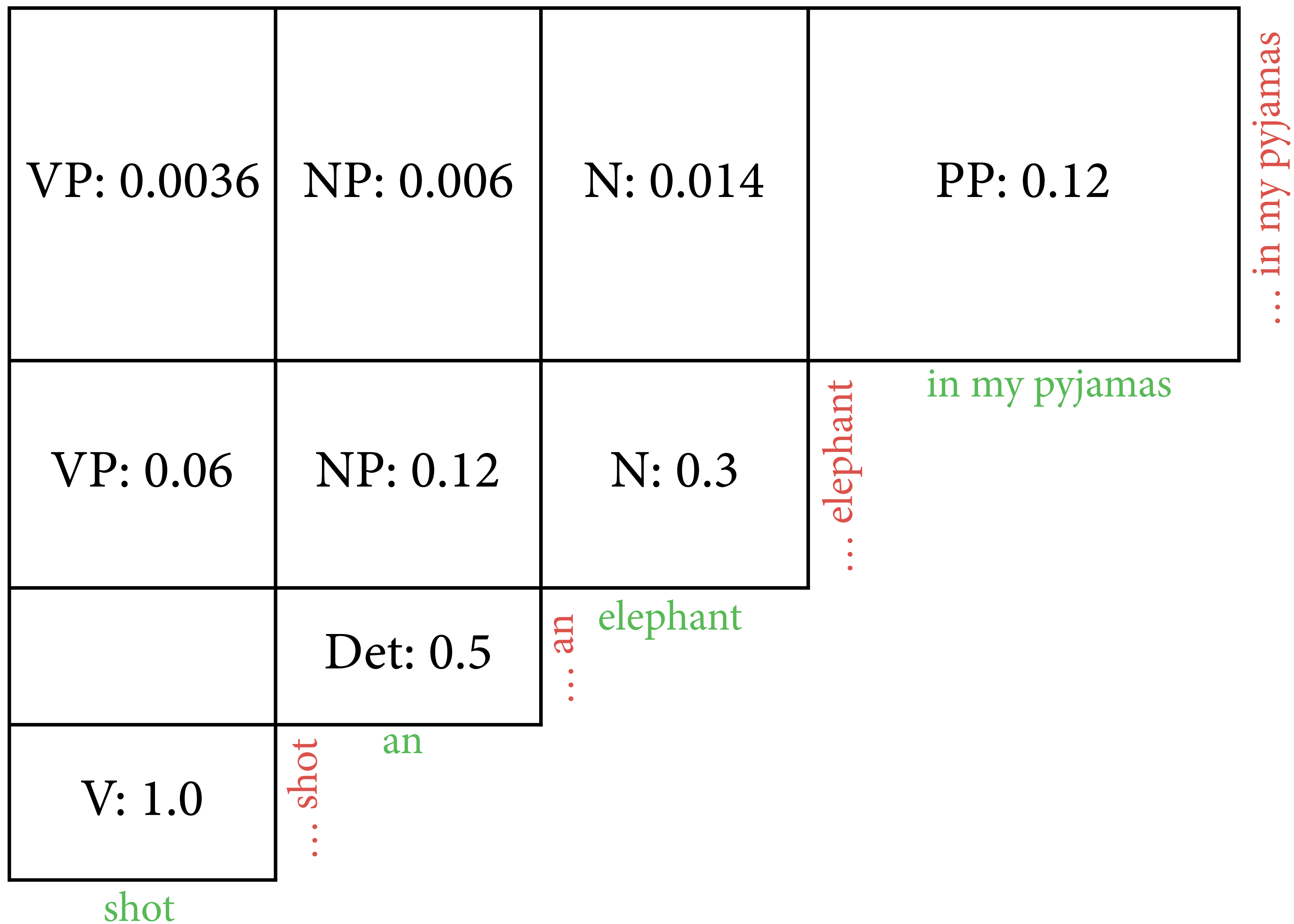
The intuition

Ordinary CKY parse chart: $Ch(i,k) = \{A \mid A \Rightarrow^* w_i \ldots w_{k-1}\}$



The intuition

Viterbi CKY parse chart: $\mathrm{Ch}(i,k) = \{(A,p) \mid p = \max_{d: A \Rightarrow^* w_i \ldots w_{k-1}} P(d)\}$



Viterbi CKY

- Define for each span (i,k) and each nonterminal A the probability

$$V(A,i,k) = \max_{A \overset{d}{\Rightarrow}^* w_i \ldots w_{k-1}} P(d)$$

- Compute V iteratively “bottom up”, i.e. starting from small spans and working our way up to longer spans.

$$V(A,i,i+1) = P(A \to w_i)$$

$$V(A,i,k) = \max_{\substack{A \to B\ C \\ i<j<k}} P(A \to B\ C) \cdot V(B,i,j) \cdot V(C,j,k)$$

Viterbi CKY - pseudocode

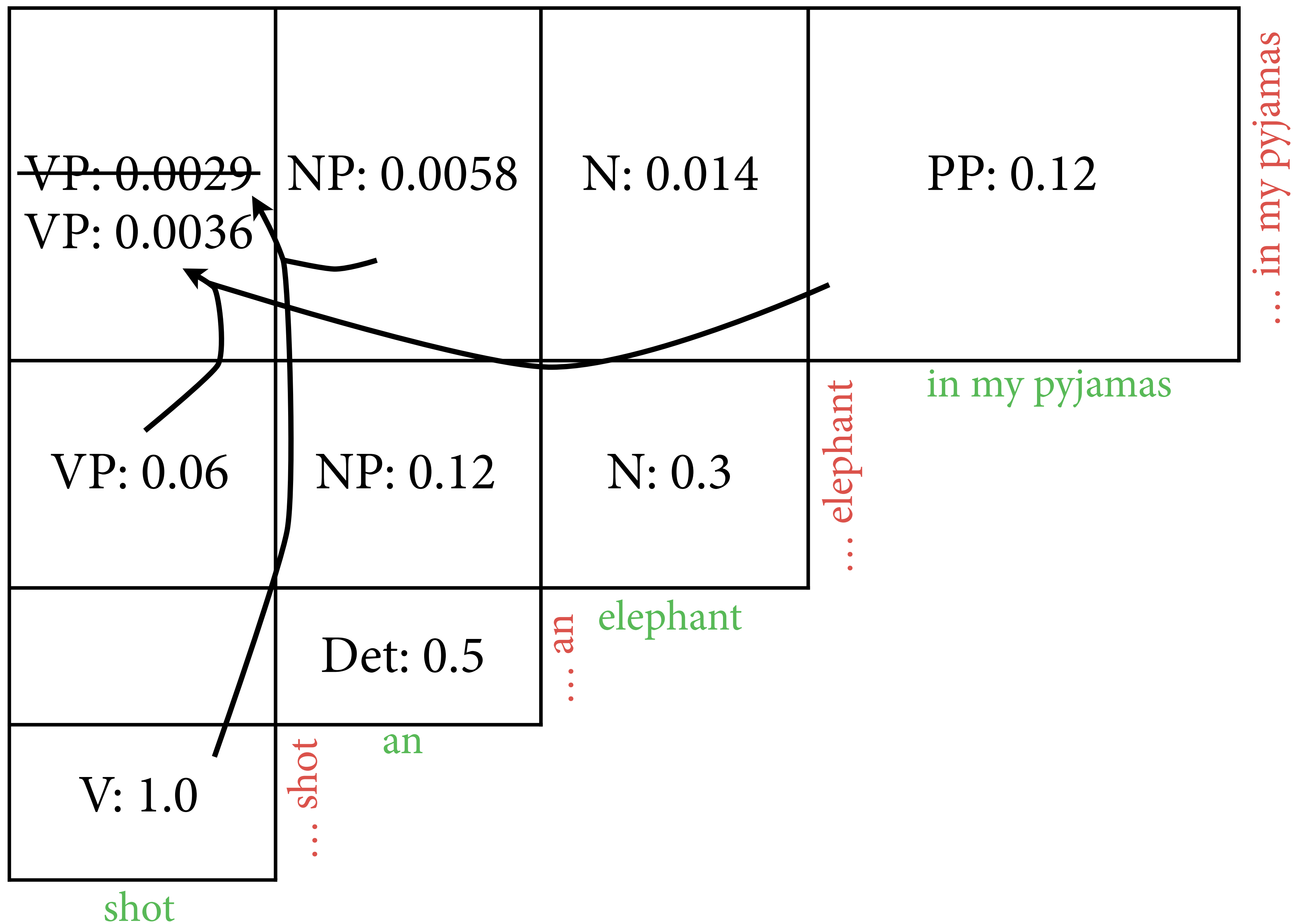
```
set all V[A,i,j] to 0

for all i from 1 to n:
    for all A with rule A -> wi:
        add A to Ch(i,i+1)
        V[A,i,i+1] = P(A -> wi)

for all b from 2 to n:
  for all i from 1 to n-b+1:
    for all k from 1 to b-1:
      for all B in Ch(i,i+k) and C in Ch(i+k,i+b):
        for all production rules A -> B C:
          add A to Ch(i,i+b)
          if P(A -> B C) * V[B,i,i+k] * V[C,i+k,i+b] > V[A,i,i+b]:
            V[A,i,i+b] = P(A -> B C) * V[B,i,i+k] * V[C,i+k,i+b]
```

Viterbi-CKY in action

Viterbi CKY parse chart: $\mathrm{Ch}(i,k) = \{(A,p) \mid p = \max_{d: A \Rightarrow^* w_i \ldots w_{k-1}} P(d)\}$



Remarks

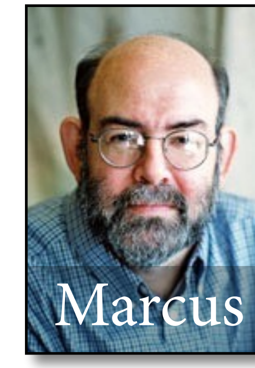
- Viterbi CKY has exactly the same nested loops as the ordinary CKY parser.
  - computing V in addition to Ch only changes constant factor
  - thus asymptotic runtime remains $O(n^3)$
- Compute optimal parse by storing backpointers.
  - same backpointers as in ordinary CKY
  - sufficient to store the *best* backpointer for each (A,i,k) if we only care about best parse (and not all parses), i.e. actually uses less memory than ordinary CKY

Obtaining the PCFG

- How to obtain the CFG?
  - write by hand
  - derive from *treebank*
  - *grammar induction* from raw text

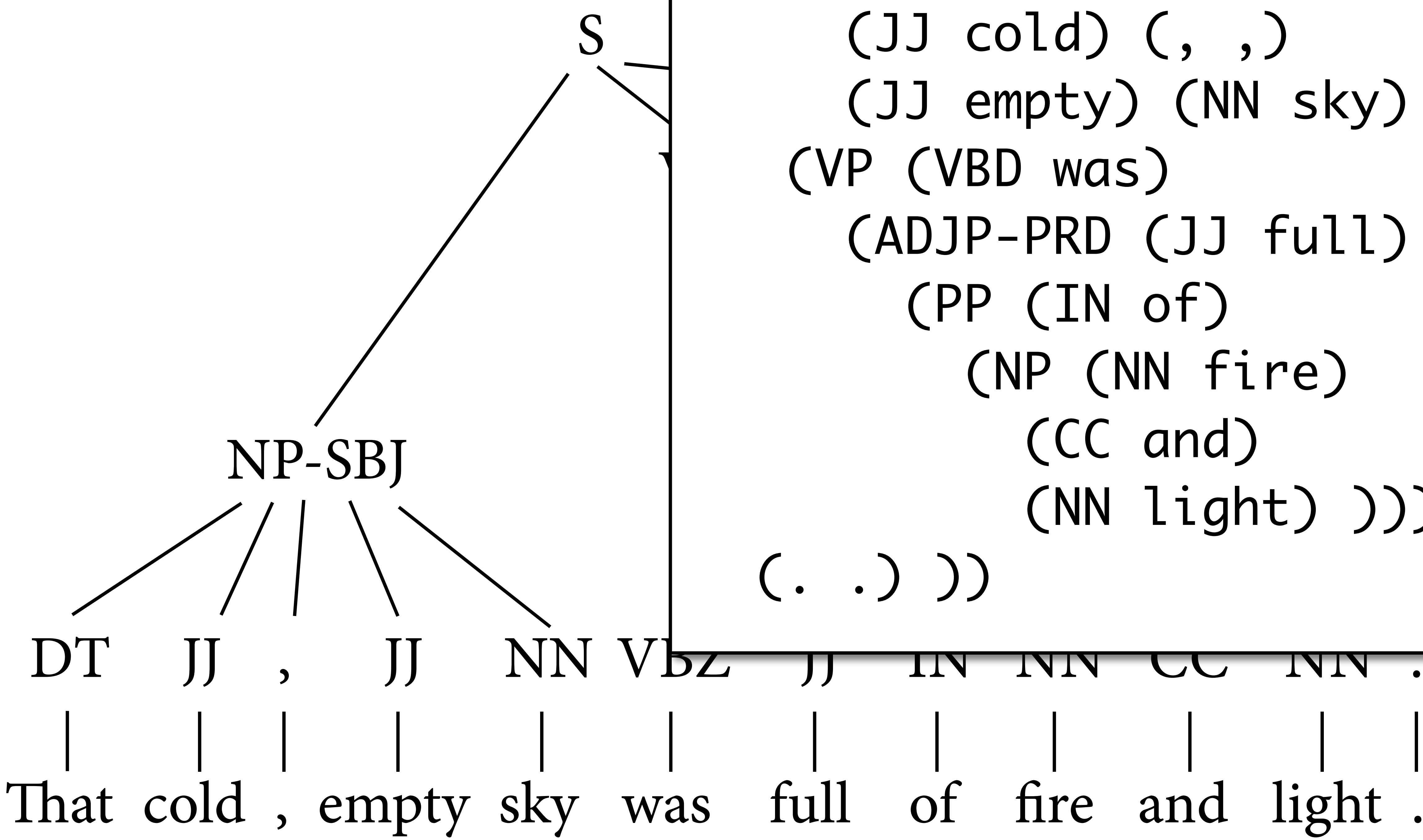
- How to obtain the rule probabilities once we have the CFG?
  - maximum likelihood estimation from treebank
  - EM training from raw text (inside-outside algorithm)

The Penn Treebank



- Large (in the mid-90s) quantity of text, annotated with POS tags and syntactic structures.
- Consists of several sub-corpora:
  - Wall Street Journal: 1 year of news text, 1 million words
  - Brown corpus: balanced corpus, 1 million words
  - ATIS: dialogues on flight bookings, 5000 words
  - Switchboard: spoken dialogue, 3 million words
- WSJ PTB is standard corpus for training and evaluating PCFG parsers.

Annotation format




```
((S
  (NP-SBJ (DT That)
    (JJ cold) (, ,)
    (JJ empty) (NN sky) )
  (VP (VBD was)
    (ADJP-PRD (JJ full)
      (PP (IN of)
        (NP (NN fire)
          (CC and)
          (NN light) ))))
 (. .) ))
```

Reading off grammar

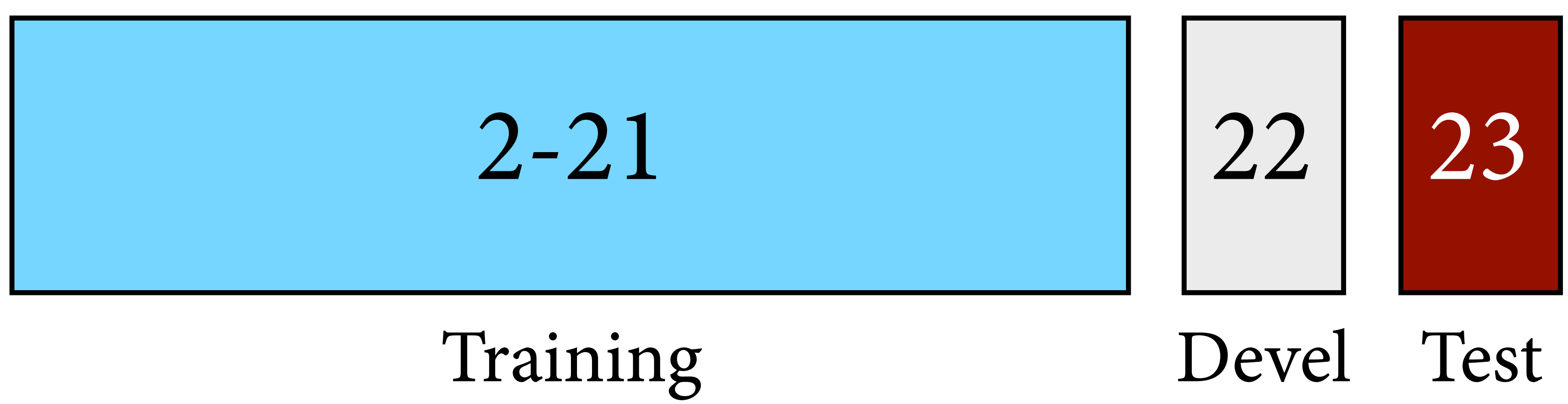
- Can directly read off "grammar in annotators' heads" from trees in treebank.

- Yields very large CFG, e.g. 4500 rules for VP:
  VP → VBD PP
  VP → VBD PP P
  VP → VBD PP P
  VP → VBD PP P
  VP → VBD ADVP PP
  VP → VBD PP ADVP
  …
  VP → VBD PP PP PP PP PP ADVP PP

> "This mostly happens because we go from football in the fall to lifting in the winter to football again in the spring."

Evaluation

- Step 1: Decide on training and test corpus.
  For WSJ corpus, there is a conventional split by sections:

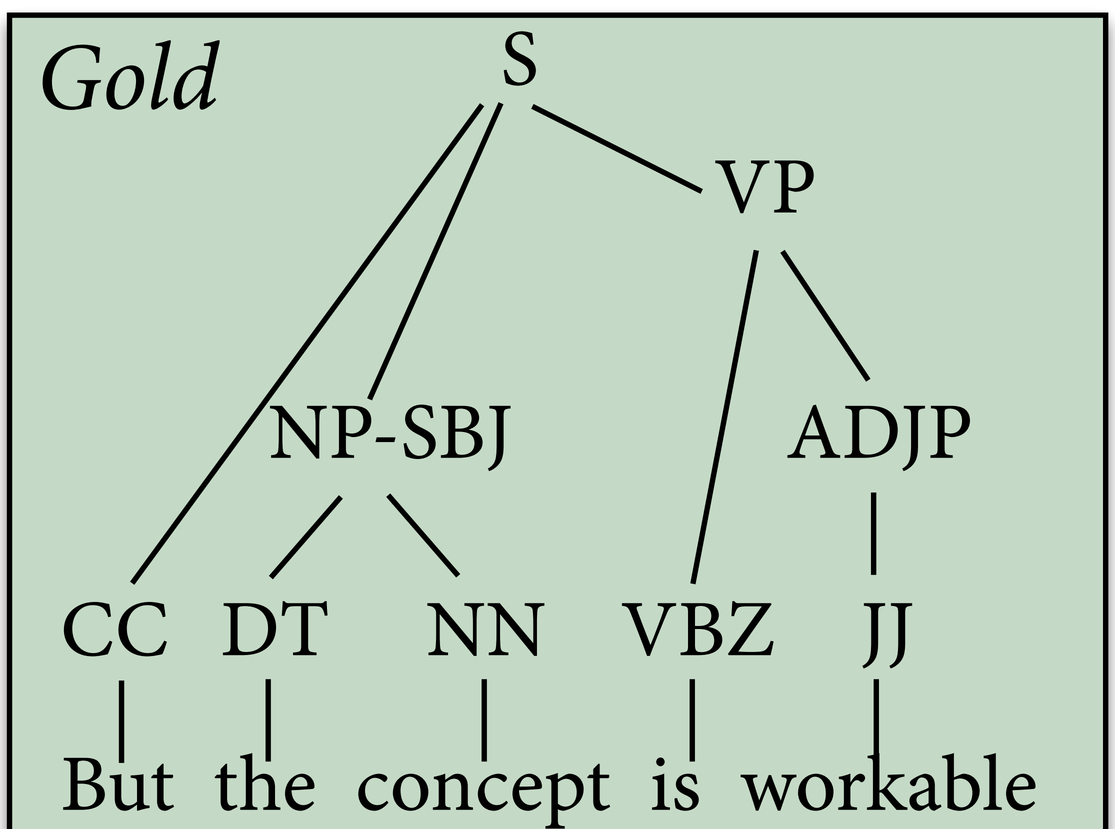


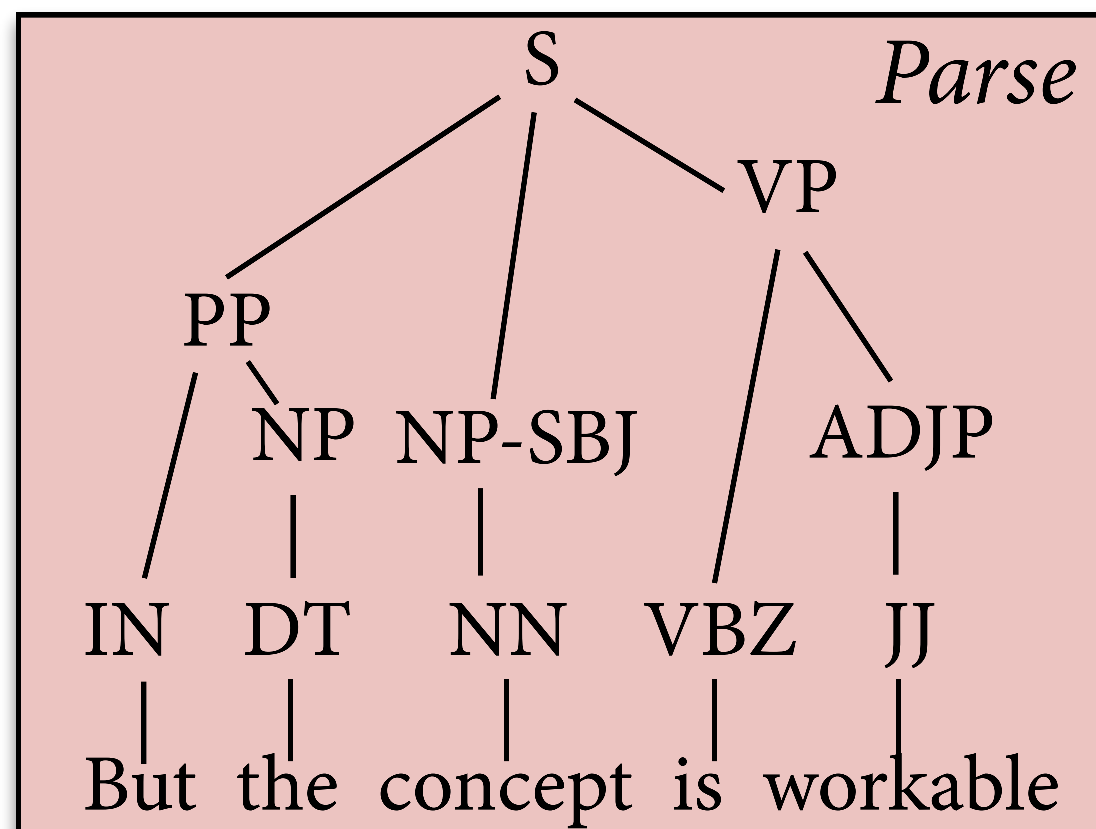
Evaluation

- Step 2: How should we measure the accuracy of the parser?
- Straightforward idea: Measure "exact match", i.e. proportion of gold standard trees that parser got right.
- This is too strict:
  - parser makes many decisions in parsing a sentence
  - a single incorrect parsing decision makes tree "wrong"
  - want more fine-grained measure

Comparing parse trees

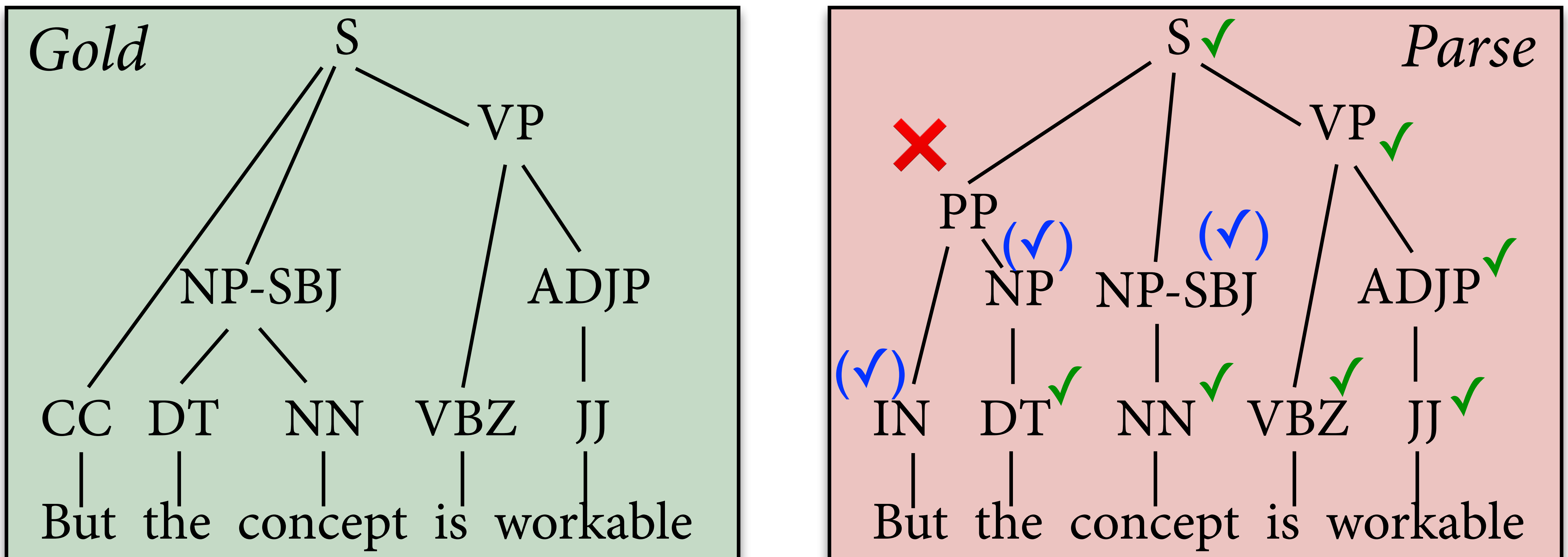
- Idea 2 (PARSEVAL): Compare *structure* of parse tree and gold standard tree.
  - Labeled: Which *constituents* (span + syntactic category) of one tree also occur in the other?
  - Unlabeled: How do the trees bracket the *substrings* of the sentence (ignoring syntactic categories)?





Precision

What proportion of constituents in *parse tree* is also present in *gold tree?*

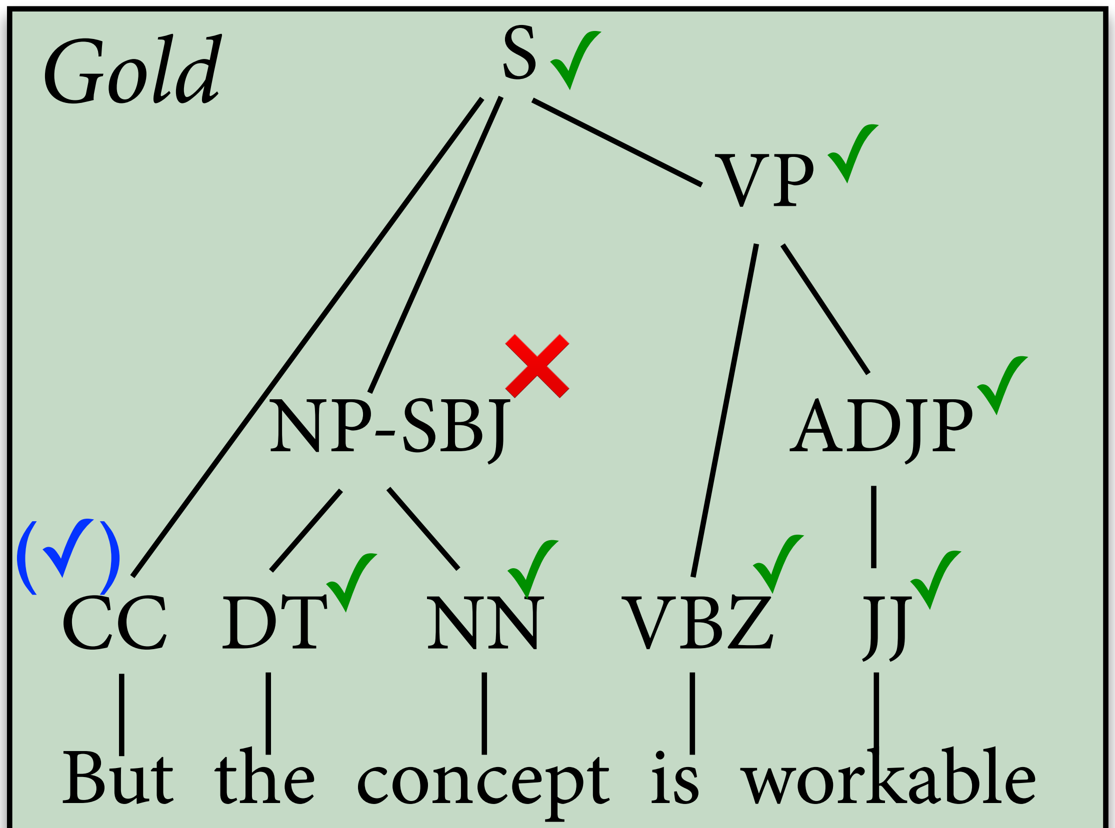


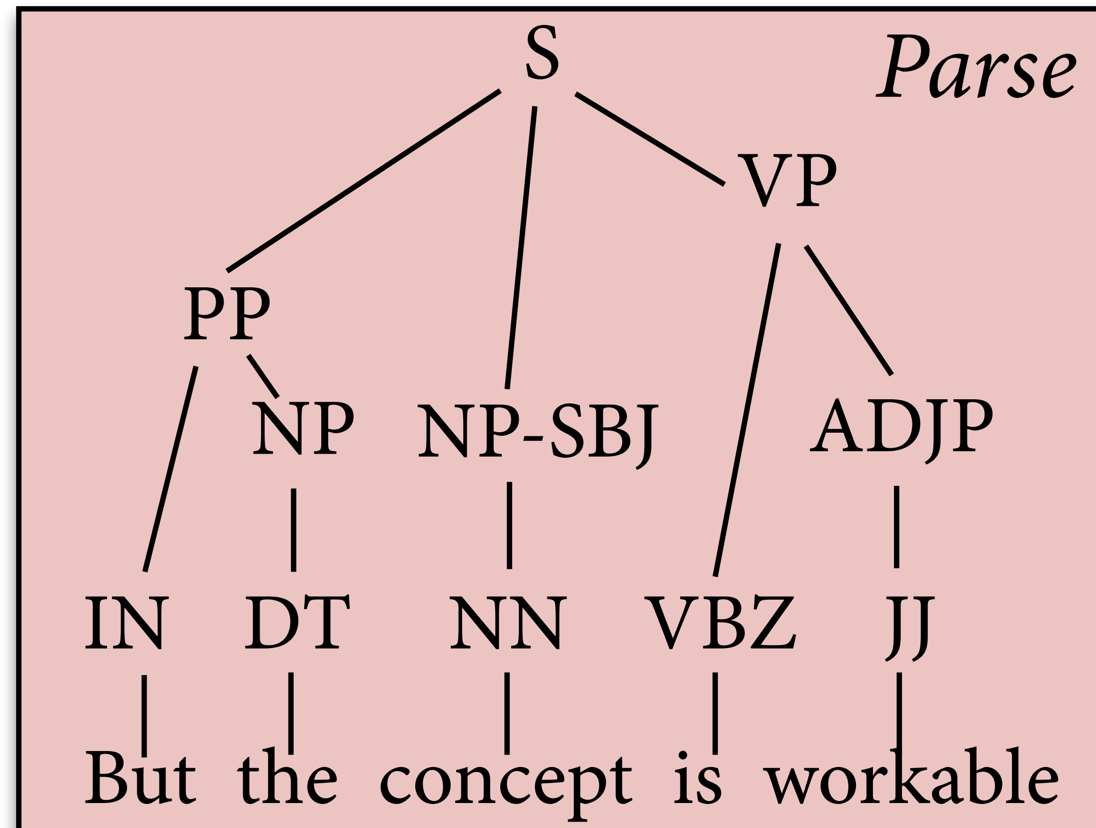
Labeled Precision = 7 / 11 = 63.6%

Unlabeled Precision = 10 / 11 = 90.9%

Recall

What proportion of constituents in *gold tree* is also present in *parse tree?*





Labeled Recall = 7 / 9 = 77.8%
Unlabeled Recall = 8 / 9 = 88.9%

F-Score

- Precision and recall measure opposing qualities of a parser ("soundness" and "completeness")

- Summarize both together in the *f-score:*

$$F_1 = \frac{2 \cdot P \cdot R}{P + R}$$

- In the example, we have labeled f-score 70.0 and unlabeled f-score 89.9.

Summary

- PCFGs extend CFGs with rule probabilities.
  - Events of random process are nonterminal expansion steps. These are all statistically independent.
  - Use Viterbi CKY parser to find most probable parse tree for a sentence in cubic time.
- Read grammars off treebanks.
  - next time: learn rule probabilities
- Evaluation of statistical parsers.