# **Training PCFGs**

Computational Linguistics

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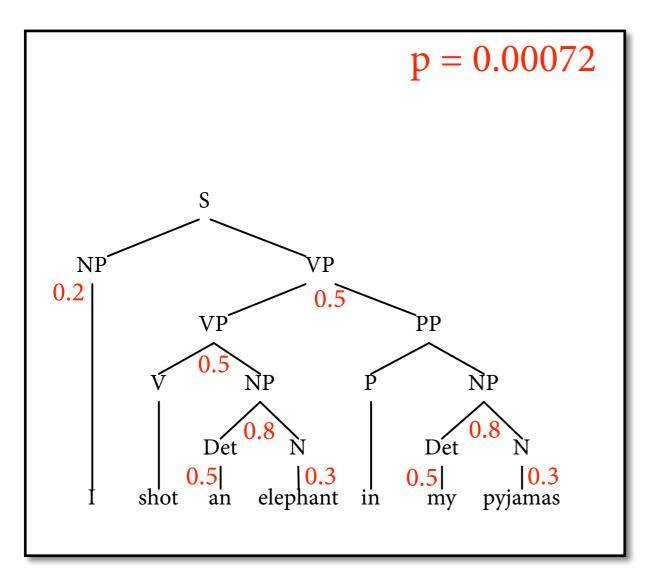
30 November 2018

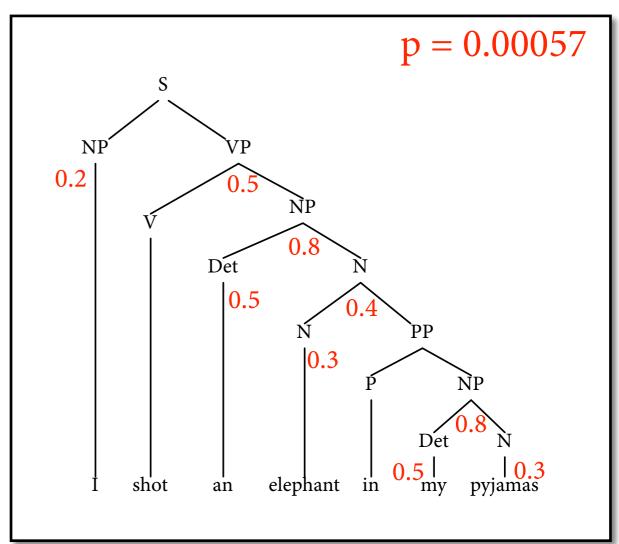
### **Probabilistic CFGs**

$S \rightarrow NP VP$	[1.0]	$VP \rightarrow V NP$	[0.5]
$NP \rightarrow Det N$	[0.8]	$VP \rightarrow VP PP$	[0.5]
$NP \rightarrow i$	[0.2]	$V \rightarrow shot$	[1.0]
$N \rightarrow N PP$	[0.4]	$PP \rightarrow P NP$	[1.0]
N → elephant	[0.3]	$P \rightarrow in$	[1.0]
N → pyjamas	[0.3]	Det → an	[0.5]
		$Det \rightarrow my$	[0.5]

(let's pretend for simplicity that Det = PRP\$)

#### Parse trees

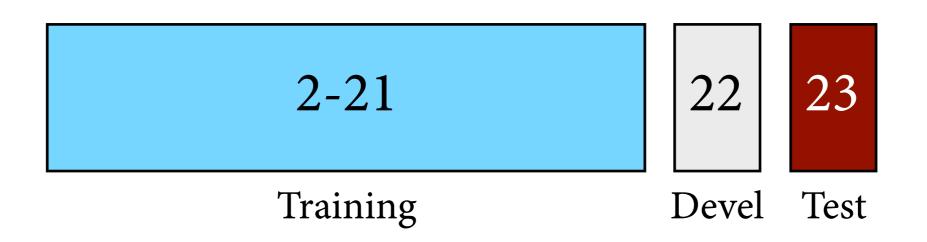




"correct" = more probable parse tree

### **Evaluation**

• Step 1: Decide on training and test corpus. For WSJ corpus, there is a conventional split by sections:

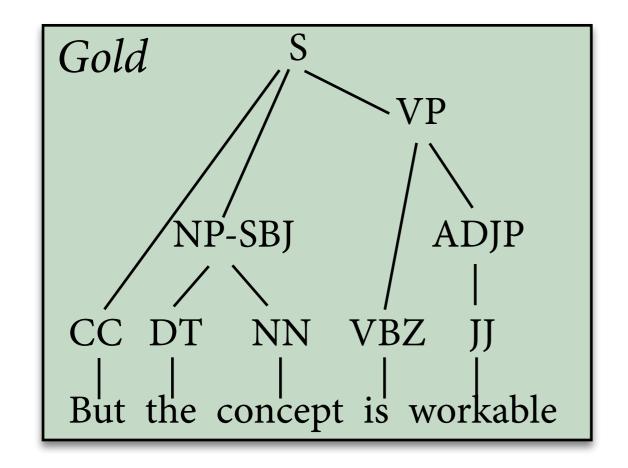


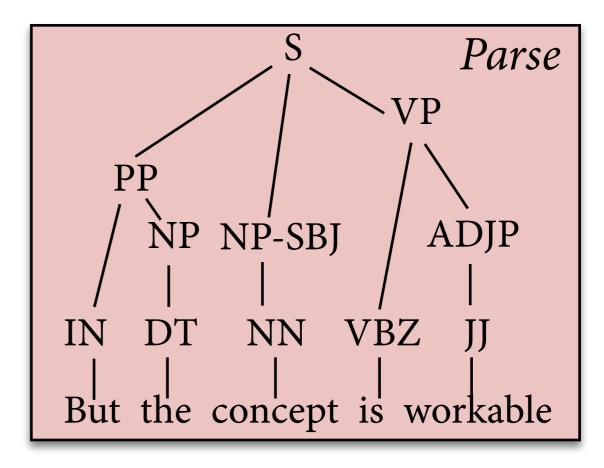
## **Evaluation**

- Step 2: How should we measure the accuracy of the parser?
- Straightforward idea: Measure "exact match", i.e. proportion of gold standard trees that parser got right.
- This is too strict:
  - parser makes many decisions in parsing a sentence
  - a single incorrect parsing decision makes tree "wrong"
  - want more fine-grained measure

# Comparing parse trees

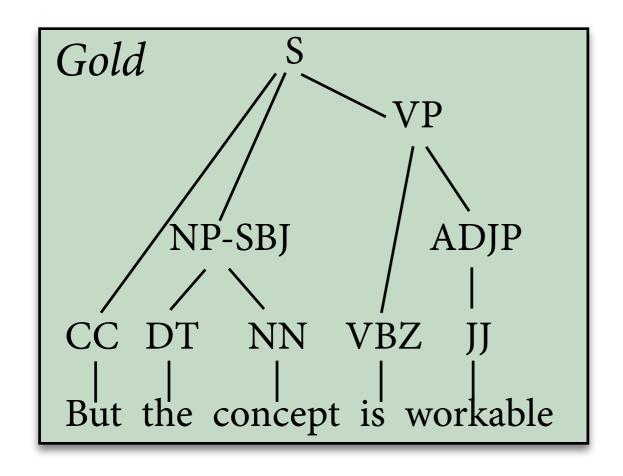
- Idea 2 (PARSEVAL): Compare *structure* of parse tree and gold standard tree.
  - ▶ Labeled: Which *constituents* (span + syntactic category) of one tree also occur in the other?
  - ▶ Unlabeled: How do the trees bracket the *substrings* of the sentence (ignoring syntactic categories)?

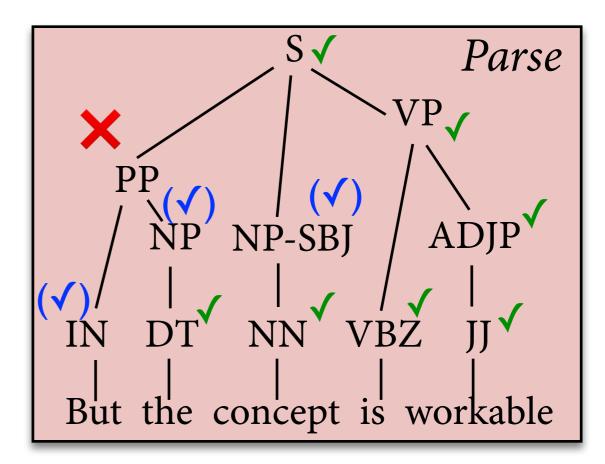




#### Precision

What proportion of constituents in *parse tree* is also present in *gold tree*?

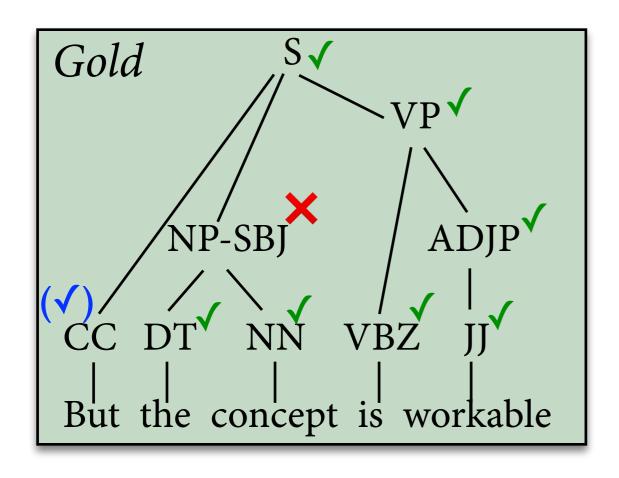


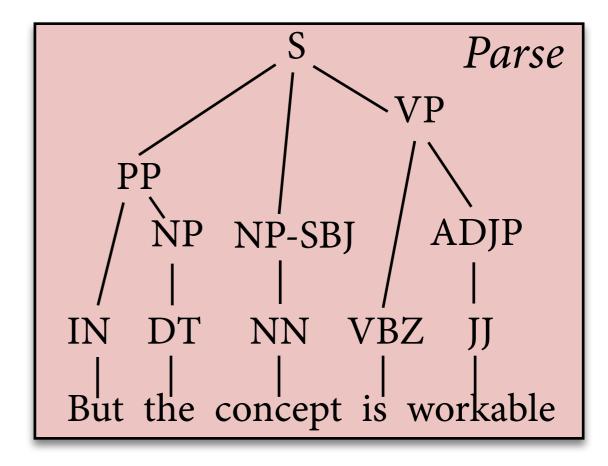


Labeled Precision = 7 / 11 = 63.6%Unlabeled Precision = 10 / 11 = 90.9%

#### Recall

What proportion of constituents in *gold tree* is also present in *parse tree*?





Labeled Recall = 7 / 9 = 77.8% Unlabeled Recall = 8 / 9 = 88.9%

## F-Score

- Precision and recall measure opposing qualities of a parser ("soundness" and "completeness")
- Summarize both together in the *f-score*:

$$F_1 = \frac{2 \cdot P \cdot R}{P + R}$$

• In the example, we have labeled f-score 70.0 and unlabeled f-score 89.9.

# Today

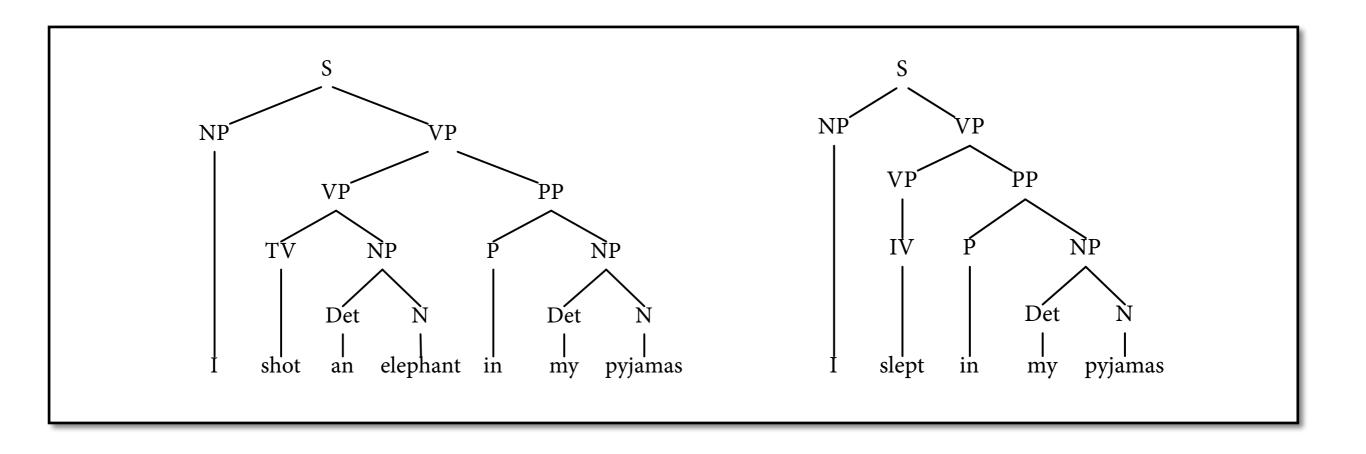
- Parameters of PCFG = rule probabilities.
- How do we learn parameters from corpora?
  - maximum likelihood estimation
  - "hard EM" using Viterbi
  - "soft EM" using the inside-outside algorithm

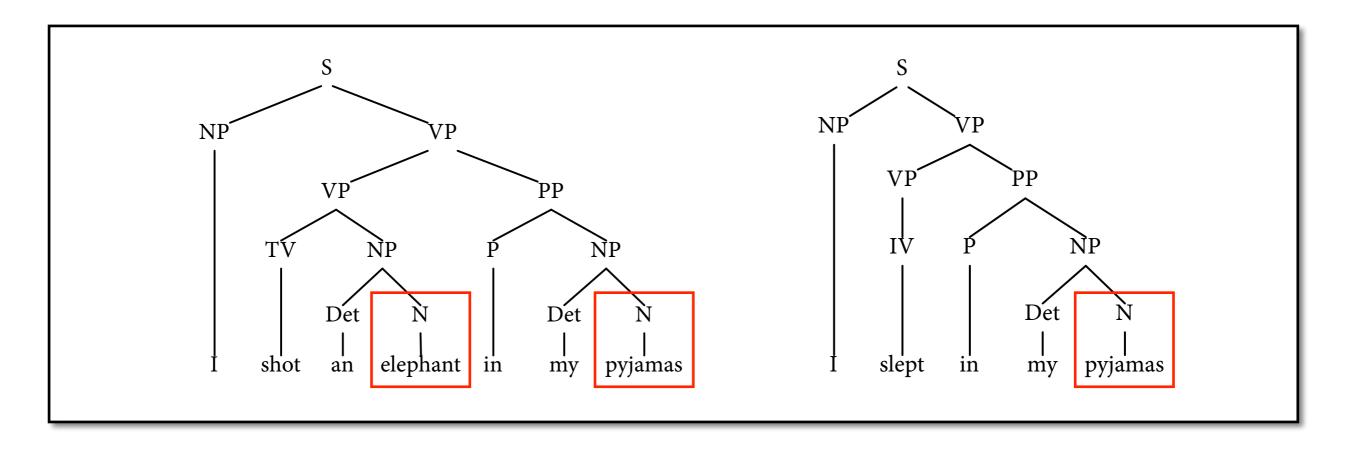
### **ML Estimation**

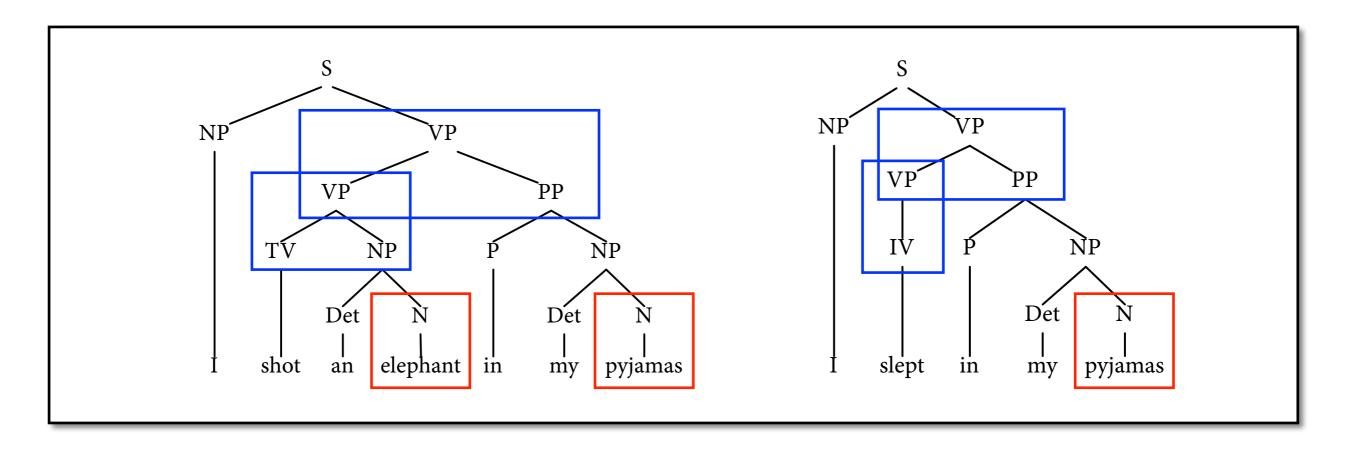
- Assume we have a treebank.
  - that is, every sentence annotated by hand with its "correct" parse tree
- Then we can use MLE to obtain rule probabilities:

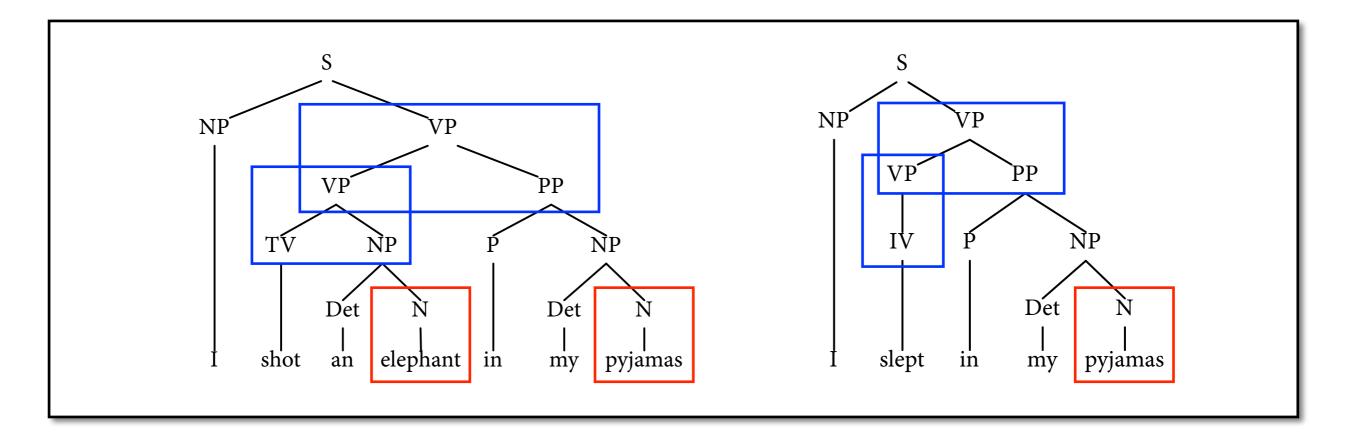
$$P(A \to w) = \frac{C(A \to w)}{C(A \to \bullet)} = \frac{C(A \to w)}{\sum_{w'} C(A \to w')}$$

Standard way of parameter estimation in practice.
 Works well, smoothing only needed for unknown words (or replace by POS tags).









$N \rightarrow N PP$	[0]	$VP \rightarrow TV NP$	[1/4]
N → elephant	[1/3]	$VP \rightarrow IV$	[1/4]
N → pyjamas	[2/3]	$VP \rightarrow VP PP$	[1/2]

## Unsupervised estimation

- MLE works okay for English.
  - German: Tiger treebank exists, but is hard for PCFGs,
     e.g. because of free word order.
  - ▶ most other languages: phrase structure annotations unavailable, expensive to create → unsupervised methods?
- Unsupervised methods:
  - provide CFG, learn parameters from unannotated corpus
  - ▶ show first "hard EM", then "soft EM"
  - ideas instructive and generalize to other problems

## "Hard" aka Viterbi EM

 In the absence of syntactic annotations, learner must invent its own parse trees.

#### • Viterbi EM:

- start with some parameter estimate
- produce "syntactic annotations" by computing best tree for each sentence using Viterbi
- apply MLE to re-estimate parameters
- repeat as long as needed
- This is *not* real EM!

1

 $N \rightarrow N PP$  [0.6]

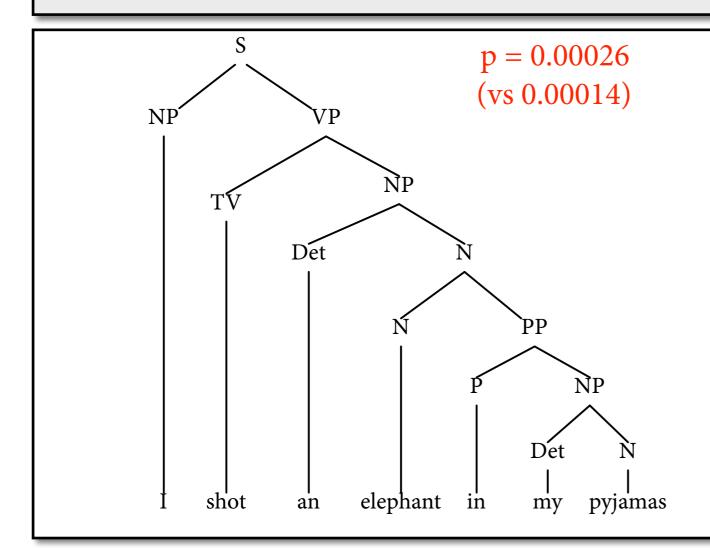
 $VP \rightarrow TV NP$  [1/3]

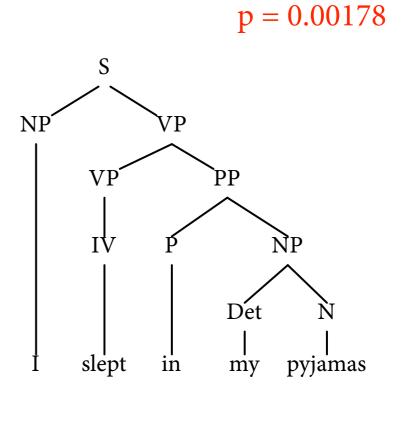
 $N \rightarrow elephant$  [0.2]

 $VP \rightarrow IV$  [1/3]

 $N \rightarrow pyjamas$  [0.2]

 $VP \rightarrow VP PP$  [1/3]





1

 $N \rightarrow N PP$  [0.6] VP

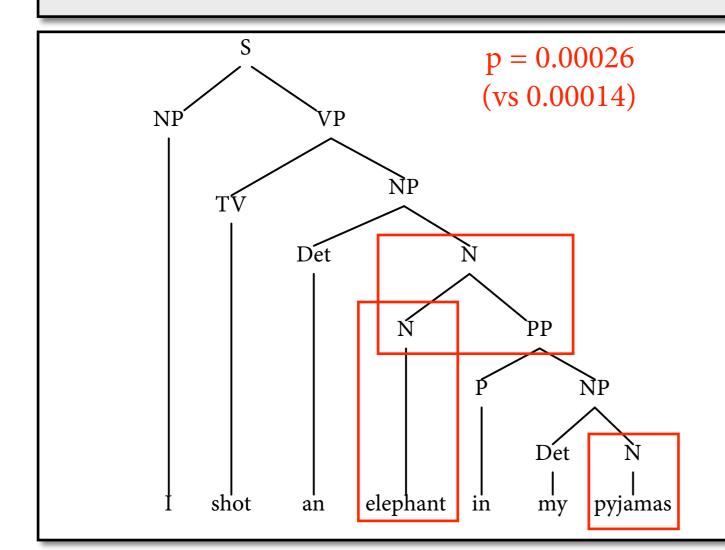
 $VP \rightarrow TV NP$  [1/3]

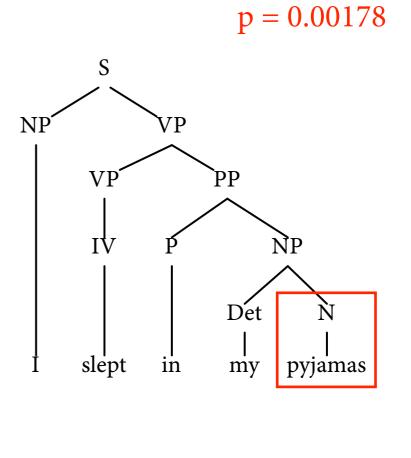
 $N \rightarrow elephant$  [0.2]

 $VP \rightarrow IV$  [1/3]

 $N \rightarrow pyjamas$  [0.2]

 $VP \rightarrow VP PP$  [1/3]





1

 $N \rightarrow N PP$  [0.6]

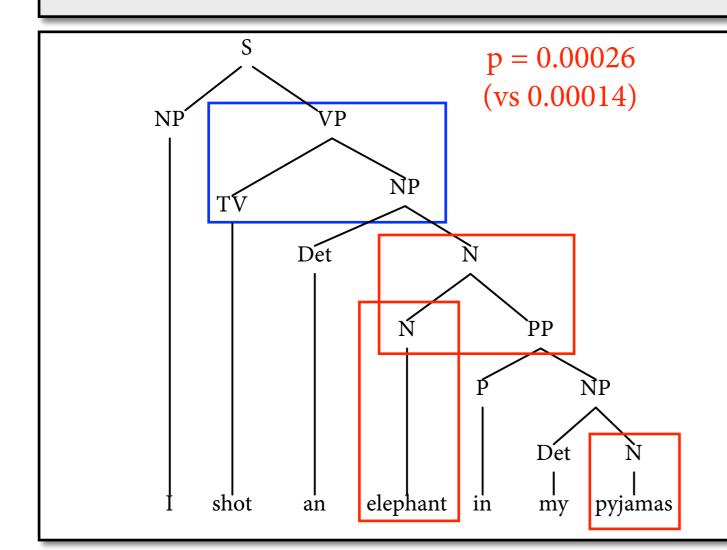
 $VP \rightarrow TV NP$  [1/3]

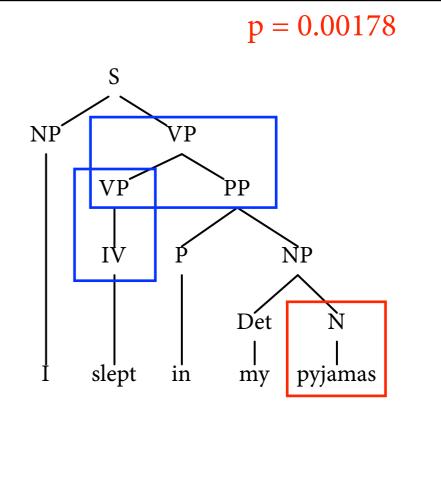
 $N \rightarrow elephant$  [0.2]

 $VP \rightarrow IV$  [1/3]

 $N \rightarrow pyjamas$  [0.2]

 $VP \rightarrow VP PP$  [1/3]





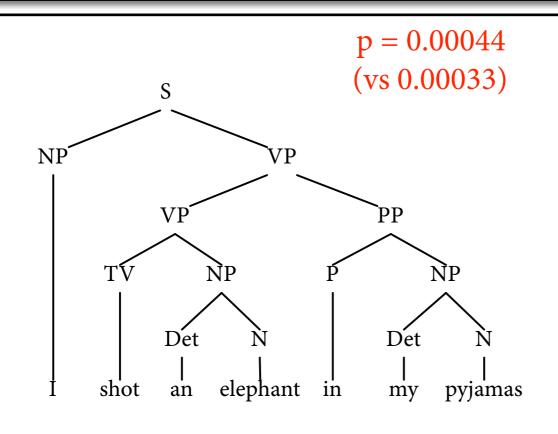
## MLE on Viterbi parses

2

```
N \rightarrow N PP [1/4] VP \rightarrow TV NP [1/3]
```

$$N \rightarrow \text{elephant}$$
 [1/4]  $VP \rightarrow IV$  [1/3]

$$N \rightarrow pyjamas$$
 [1/2]  $VP \rightarrow VP PP$  [1/3]



S NP VP PP IV P NP Det N I slept in my pyjamas

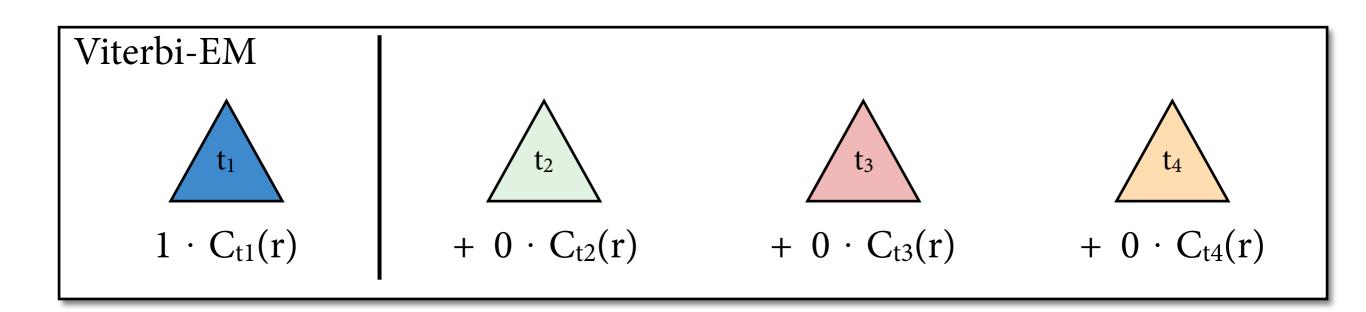
p = 0.00889

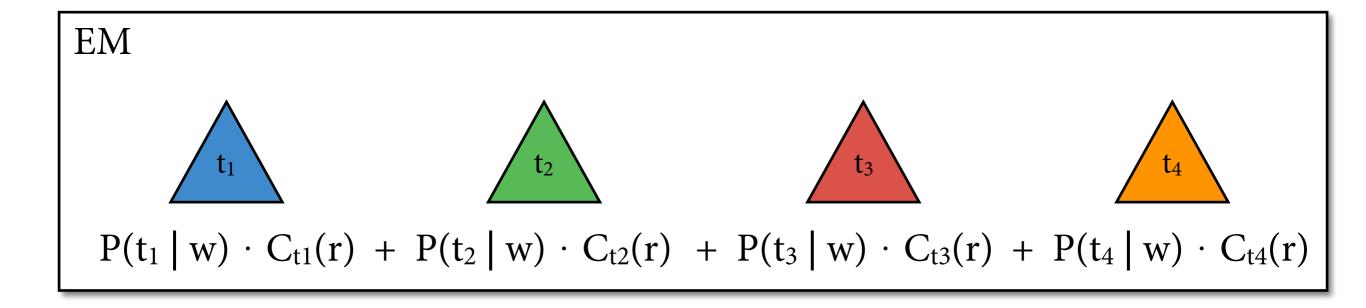
# Some things to note

- In this example, the likelihood increased.
  - ▶ this need not always be the case for Viterbi EM
- Viterbi EM commits to a single parse tree per sentence. This has advantages and disadvantages:
  - parse tree easy to compute, and can simply apply MLE
  - ignores all uncertainty we had about correct parse (winning parse tree takes all)

## Towards "real" (aka "soft")

idea: weighted counting of rules in all parse trees





# **Expected counts**

 Define *expected count* of rule A → B C, based on previous parameter estimate.

$$E(A \to B \ C) = \sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_t(A \to B \ C)$$

• If we have them, can re-estimate parameters:

$$P(A \to B \ C) = \frac{E(A \to B \ C)}{\sum_{r} E(A \to r)}$$

- Challenge: How to compute  $E(A \rightarrow B C)$  efficiently?
  - we assume grammars in CNF here

### Fundamental idea

$$E(A \to B \ C) = \sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_t(A \to B \ C)$$

$$= \frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot C_t(A \to B \ C)$$

$$= \frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot \sum_{i,j,k} ||\text{rule for } i, j, k \text{ in } t \text{ is } A \to B \ C||$$

$$= \frac{1}{P(w)} \sum_{i,j,k} \left( \sum_{t \in \mathcal{T}} P(t) \cdot ||\text{rule for } i, j, k \text{ in } t \text{ is } A \to B \ C|| \right)$$

$$= \frac{1}{P(w)} \sum_{i,j,k} \left( \sum_{t \text{ of this form}} P(t) \right)$$
The that  $P(t, w) = P(t)$ 

(note that P(t, w) = P(t))

### Fundamental idea

$$\begin{split} E(A \to B \ C) &= \sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_t(A \to B \ C) \\ &= \frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot C_t(A \to B \ C) \\ &= \frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot \sum_{i,j,k} || \text{rule for } i,j,k \text{ in } t \text{ is } A \to B \ C|| \\ &= \frac{1}{P(w)} \sum_{i,j,k} \left( \sum_{t \in \mathcal{T}} P(t) \cdot || \text{rule for } i,j,k \text{ in } t \text{ is } A \to B \ C|| \right) \\ &= \frac{1}{P(w)} \sum_{i,j,k} \left( \sum_{t \in \mathcal{T}} P(t) \cdot || \text{rule for } i,j,k \text{ in } t \text{ is } A \to B \ C|| \right) \\ &= \text{call this term } \mu(A \to B \ C, i,j,k) \end{split}$$

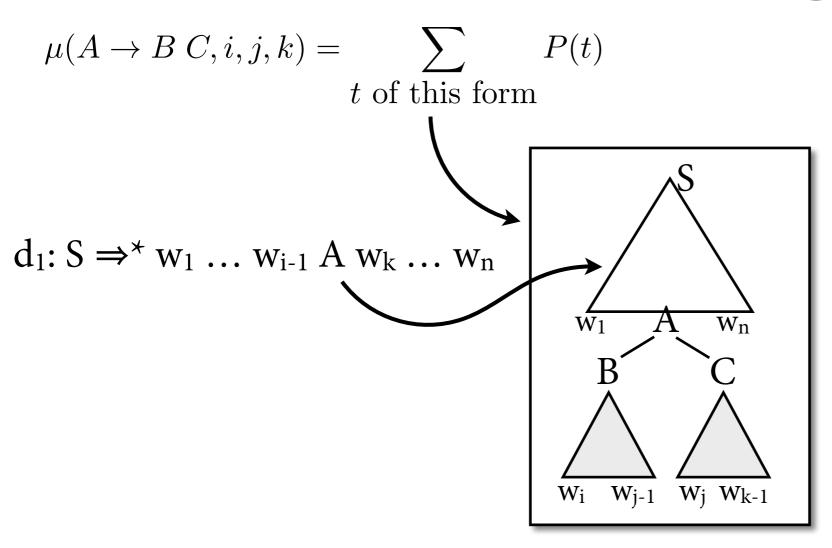
 $W_{j-1}$ 

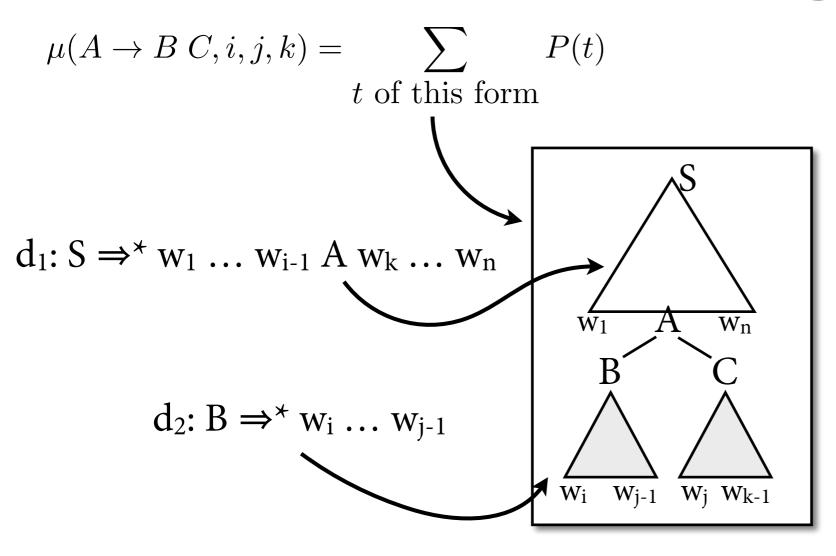
 $W_i$   $W_{k-1}$ 

(note that P(t, w) = P(t))

 $\overline{W_{j-1}}$   $\overline{W_j}$   $W_{k-1}$ 

$$\mu(A \to B \ C, i, j, k) = \sum_{t \text{ of this form}} P(t)$$





$$\mu(A \to B \ C, i, j, k) = \sum_{t \text{ of this form}} P(t)$$

$$d_1: S \Rightarrow^* w_1 \dots w_{i-1} A w_k \dots w_n$$

$$d_2: B \Rightarrow^* w_i \dots w_{j-1}$$

$$d_3: C \Rightarrow^* w_j \dots w_{k-1}$$

$$\mu(A \to B \ C, i, j, k) = \sum_{t \text{ of this form}} P(t)$$

$$d_1: S \Rightarrow^* w_1 \dots w_{i-1} A w_k \dots w_n$$

$$d_2: B \Rightarrow^* w_i \dots w_{j-1}$$

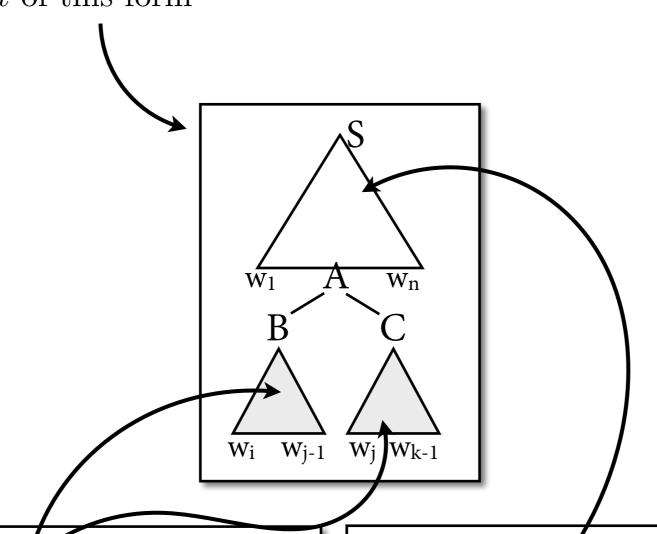
$$d_3: C \Rightarrow^* w_j \dots w_{k-1}$$

$$\mu(A \to B \ C, i, j, k) = \sum_{t \text{ of this form}} P(t)$$

$$= \sum_{t \text{ of this form}} P(d_1) \cdot P(A \to B \ C) \cdot P(d_2) \cdot P(d_3)$$

$$= \left(\sum_{d_1} P(d_1)\right) \cdot P(A \to B \ C) \cdot \left(\sum_{d_2} P(d_2)\right) \cdot \left(\sum_{d_3} P(d_3)\right)$$

$$\mu(A \to B \ C, i, j, k) = \sum_{\substack{t \text{ of this form}}} P(t) = \alpha(A, i, k) \cdot P(A \to B \ C) \cdot \beta(B, i, j) \cdot \beta(C, j, k)$$



inside probability

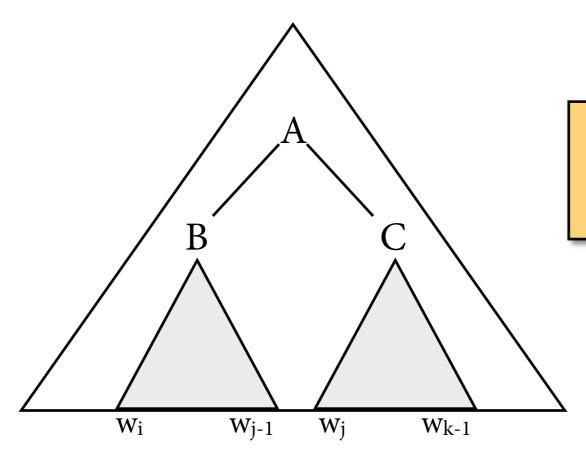
$$\beta(B, i, j) = \sum_{B \stackrel{d}{\Rightarrow}^* w_i \dots w_{j-1}} P(d)$$

outside probability

$$\alpha(A, i, k) = \sum_{S \stackrel{d}{\Longrightarrow}^* w_1 \dots w_{i-1} A w_1 \dots w_m} P(d)$$

## Inside probabilities

$$\beta(B, i, j) = \sum_{B \stackrel{d}{\Longrightarrow}^* w_i \dots w_{j-1}} P(d)$$



special case:

$$P(w) = \beta(S,1,n+1)$$

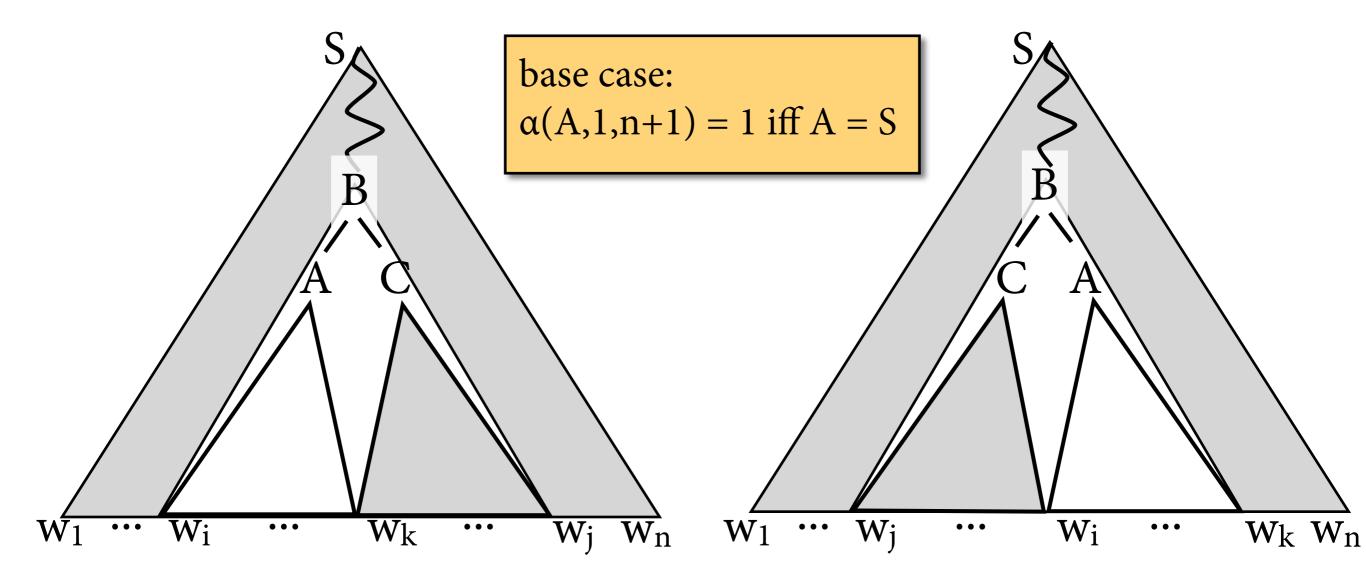
$$\beta(A, i, i + 1) = P(A \to w_i)$$

$$\beta(A, i, k) = \sum_{\substack{A \to B \ C \ i < i < k}} P(A \to B \ C) \cdot \beta(B, i, j) \cdot \beta(C, j, k)$$

## Outside probabilities

$$\alpha(A, i, k) = \sum_{S \stackrel{d}{\Longrightarrow}^* w_1 \dots w_{i-1} A w_k \dots w_n} P(d)$$

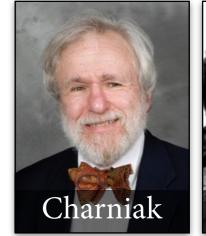
$$= \sum_{\substack{B \to A \ C \\ k < j \leq n}} P(B \to A \ C) \cdot \beta(C, k, j) \cdot \alpha(B, i, j) + \sum_{\substack{B \to C \ A \\ 1 \leq j < i}} P(B \to C \ A) \cdot \beta(C, j, i) \cdot \alpha(B, j, k)$$



# The Inside-Outside Algorithm

- Start with some initial estimate of parameters.
- For each sentence w, compute  $\alpha$ ,  $\beta$ , and  $\mu$ .
- Compute expected counts  $E(A \rightarrow B C)$ .
  - sum expected counts over all sentences
  - remember that  $P(w) = \beta(S, 1, n+1)$
- Re-estimate  $P(A \rightarrow B C)$  from expected counts.
- Iterate until convergence.

### Some remarks





- Inside-outside increases likelihood in each step.
- But huge problems with local maxima.
  - ➤ Carroll & Charniak 92 find 300 different local maxima for 300 different initial parameter estimates.
  - ▶ Improve by partially bracketing strings (Pereira & Schabes 92).
- Therefore, EM doesn't really work for totally unsupervised PCFG training.
- But extremely useful in refining existing grammars (Berkeley parser; see next time).

## Summary

- Learning parameters of PCFGs:
  - maximum likelihood estimation from raw text
  - "hard EM": iterate MLE on Viterbi parses
  - ▶ EM: use inside-outside algorithm with expected rule counts
- PCFG parsing with MLE parse gets f-score in low 70's. Will improve on this next time (state of the art: 93).
- Have assumed that CFG is given and only parameters are to be learned. Will fix this later in this course.