# Training PCFGs 

Computational Linguistics

Alexander Koller

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## Probabilistic CFGs

| $\mathrm{S} \rightarrow \mathrm{NP}$ VP | $[1.0]$ | $\mathrm{VP} \rightarrow \mathrm{V} \mathrm{NP}$ | $[0.5]$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{NP} \rightarrow$ Det N | $[0.8]$ | $\mathrm{VP} \rightarrow \mathrm{VP} P \mathrm{PP}$ | $[0.5]$ |
| $\mathrm{NP} \rightarrow \mathrm{i}$ | $[0.2]$ | $\mathrm{V} \rightarrow$ shot | $[1.0]$ |
| $\mathrm{N} \rightarrow \mathrm{N} \mathrm{PP}$ | $[0.4]$ | $\mathrm{PP} \rightarrow \mathrm{P} \mathrm{NP}$ | $[1.0]$ |
| $\mathrm{N} \rightarrow$ elephant | $[0.3]$ | $\mathrm{P} \rightarrow$ in | $[1.0]$ |
| $\mathrm{N} \rightarrow$ pyjamas | $[0.3]$ | Det $\rightarrow$ an | $[0.5]$ |
|  |  | Det $\rightarrow$ my | $[0.5]$ |

(let's pretend for simplicity that Det $=$ PRP\$)

## Parse trees



## 1

"correct" = more probable parse tree

## Evaluation

- Step 1: Decide on training and test corpus. For WSJ corpus, there is a conventional split by sections:



## Evaluation

- Step 2: How should we measure the accuracy of the parser?
- Straightforward idea: Measure "exact match", i.e. proportion of gold standard trees that parser got right.
- This is too strict:
- parser makes many decisions in parsing a sentence
- a single incorrect parsing decision makes tree "wrong"
- want more fine-grained measure


## Comparing parse trees

- Idea 2 (PARSEVAL): Compare structure of parse tree and gold standard tree.
- Labeled: Which constituents (span + syntactic category) of one tree also occur in the other?
- Unlabeled: How do the trees bracket the substrings of the sentence (ignoring syntactic categories)?



## Precision

What proportion of constituents in parse tree is also present in gold tree?


Labeled Precision $=7 / 11=63.6 \%$
Unlabeled Precision $=10 / 11=90.9 \%$

## Recall

What proportion of constituents in gold tree is also present in parse tree?


Labeled Recall $=7 / 9=77.8 \%$
Unlabeled Recall $=8 / 9=88.9 \%$

## F-Score

- Precision and recall measure opposing qualities of a parser ("soundness" and "completeness")
- Summarize both together in the $f$-score:

$$
F_{1}=\frac{2 \cdot P \cdot R}{P+R}
$$

- In the example, we have labeled f-score 70.0 and unlabeled f-score 89.9.


## Today

- Parameters of PCFG = rule probabilities.
- How do we learn parameters from corpora?
- maximum likelihood estimation
- "hard EM" using Viterbi
- "soft EM" using the inside-outside algorithm


## ML Estimation

- Assume we have a treebank.
- that is, every sentence annotated by hand with its "correct" parse tree
- Then we can use MLE to obtain rule probabilities:

$$
P(A \rightarrow w)=\frac{C(A \rightarrow w)}{C(A \rightarrow \bullet)}=\frac{C(A \rightarrow w)}{\sum_{w^{\prime}} C\left(A \rightarrow w^{\prime}\right)}
$$

- Standard way of parameter estimation in practice. Works well, smoothing only needed for unknown words (or replace by POS tags).


## Example



## Example



## Example



## Example


$\mathrm{N} \rightarrow \mathrm{N}$ PP
$\mathrm{N} \rightarrow$ elephant
$\mathrm{N} \rightarrow$ pyjamas
[2/3]
[1/3]
VP $\rightarrow$ IV
[1/4]
VP $\rightarrow$ TV NP
[1/4]
[0]
$\mathrm{VP} \rightarrow \mathrm{VP}$ PP
[1/2]

## Unsupervised estimation

- MLE works well for English.
- German: Tiger treebank exists, but is hard for PCFGs, e.g. because of free word order.
- most other languages: phrase structure annotations unavailable, expensive to create $\rightarrow$ unsupervised methods?
- Unsupervised methods:
- provide CFG, learn parameters from unannotated corpus
- show first "hard EM", then "soft EM"
- ideas instructive and generalize to related problems


## "Hard" aka Viterbi EM

- In the absence of syntactic annotations, learner must invent its own parse trees.
- Viterbi EM:
- start with some parameter estimate
- produce "syntactic annotations" by computing best tree for each sentence using Viterbi
- apply MLE to re-estimate parameters
- repeat as long as needed
- This is not real EM!


## Example

$\mathrm{N} \rightarrow \mathrm{N}$ PP
[0.6]
VP $\rightarrow$ TV NP
[1/3]
$\mathrm{N} \rightarrow$ elephant
[0.2]
$\mathrm{N} \rightarrow$ pyjamas
[0.2]
VP $\rightarrow$ IV
[1/3]
$\mathrm{VP} \rightarrow \mathrm{VP}$ PP
[1/3]


## Example

$\mathrm{N} \rightarrow \mathrm{N}$ PP
[0.6]
VP $\rightarrow$ TV NP
[1/3]
$\mathrm{N} \rightarrow$ elephant
[0.2]
$\mathrm{N} \rightarrow$ pyjamas
[0.2]
VP $\rightarrow$ IV
[1/3]
$\mathrm{VP} \rightarrow \mathrm{VP} P \mathrm{P}$
[1/3]


## Example

$$
\begin{array}{llll}
\mathrm{N} \rightarrow \mathrm{~N} \text { PP } & {[0.6]} & \mathrm{VP} \rightarrow \mathrm{TV} \text { NP } & {[1 / 3]} \\
\mathrm{N} \rightarrow \text { elephant } & {[0.2]} & \mathrm{VP} \rightarrow \mathrm{IV} & {[1 / 3]} \\
\mathrm{N} \rightarrow \text { pyjamas } & {[0.2]} & \mathrm{VP} \rightarrow \mathrm{VP} \text { PP } & {[1 / 3]}
\end{array}
$$



## MLE on Viterbi parses

$\mathrm{N} \rightarrow \mathrm{N}$ PP
[1/4]
$\mathrm{VP} \rightarrow \mathrm{TV} \mathrm{NP}$
[1/3]
$\mathrm{N} \rightarrow$ elephant
[1/4]
$\mathrm{VP} \rightarrow$ IV
[1/3]
$\mathrm{N} \rightarrow$ pyjamas
[1/2]
$\mathrm{VP} \rightarrow \mathrm{VP}$ PP
[1/3]


## Some things to note

- In this example, the likelihood increased.
- this need not always be the case for Viterbi EM
- Viterbi EM commits to a single parse tree per sentence. This has advantages and disadvantages:
- parse tree easy to compute, and can simply apply MLE
- ignores all uncertainty we had about correct parse (winning parse tree takes all)


## Towards"real" (aka"soft")

idea: weighted counting of rules in all parse trees

| Viterbi-EM |  |  |
| :---: | :---: | :---: | :---: |
| $1 \cdot \mathrm{C}_{\mathrm{t} 1}(\mathrm{r})$ |  |  |
| $+0 \cdot \mathrm{C}_{\mathrm{t} 2}(\mathrm{r})$ | $+0 \cdot \mathrm{C}_{\mathrm{t} 3}(\mathrm{r})$ | $+0 \cdot \mathrm{C}_{\mathrm{t} 4}(\mathrm{r})$ |

## EM



$$
\mathrm{P}\left(\mathrm{t}_{1} \mid \mathrm{w}\right) \cdot \mathrm{C}_{\mathrm{t} 1}(\mathrm{r})+\mathrm{P}\left(\mathrm{t}_{2} \mid \mathrm{w}\right) \cdot \mathrm{C}_{\mathrm{t} 2}(\mathrm{r})+\mathrm{P}\left(\mathrm{t}_{3} \mid \mathrm{w}\right) \cdot \mathrm{C}_{\mathrm{t} 3}(\mathrm{r})+\mathrm{P}\left(\mathrm{t}_{4} \mid \mathrm{w}\right) \cdot \mathrm{C}_{\mathrm{t} 4}(\mathrm{r})
$$

## Expected counts

- Define expected count of rule A $\rightarrow$ B C, based on previous parameter estimate.

$$
E(A \rightarrow B C)=\sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_{t}(A \rightarrow B C)
$$

- If we have them, can re-estimate parameters:

$$
P(A \rightarrow B C)=\frac{E(A \rightarrow B C)}{\sum_{r} E(A \rightarrow r)}
$$

- Challenge: How to compute $\mathrm{E}(\mathrm{A} \rightarrow \mathrm{B} \mathrm{C})$ efficiently?
- we assume grammars in CNF here


## Fundamental idea

$$
\begin{aligned}
E(A \rightarrow B C) & =\sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_{t}(A \rightarrow B C) \\
& =\frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot C_{t}(A \rightarrow B C) \\
& =\frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot \sum_{i, j, k} \| \text { rule for } i, j, k \text { in } t \text { is } A \rightarrow B C \| \\
& =\frac{1}{P(w)} \sum_{i, j, k}\left(\sum_{t \in \mathcal{T}} P(t) \cdot \| \text { rule for } i, j, k \text { in } t \text { is } A \rightarrow B C \|\right) \\
& =\frac{1}{P(w)} \sum_{i, j, k}\left(\sum_{t} P(t)\right) \\
\text { (note this form } \mathrm{P}(\mathrm{t}, \mathrm{w}) & =\mathrm{P}(\mathrm{t}))
\end{aligned}
$$

## Fundamental idea

$$
\begin{aligned}
E(A \rightarrow B C) & =\sum_{t \in \mathcal{T}} P(t \mid w) \cdot C_{t}(A \rightarrow B C) \\
& =\frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot C_{t}(A \rightarrow B C) \\
& =\frac{1}{P(w)} \sum_{t \in \mathcal{T}} P(t) \cdot \sum_{i, j, k} \| \text { rule for } i, j, k \text { in } t \text { is } A \rightarrow B C \| \\
& =\frac{1}{P(w)} \sum_{i, j, k}\left(\sum_{t \in \mathcal{T}} P(t) \cdot \| \text { rule for } i, j, k \text { in } t \text { is } A \rightarrow B C \|\right) \\
& =\frac{1}{P(w)} \sum_{i, j, k} \underbrace{}_{\text {call this term } \mu(\mathrm{A} \rightarrow \mathrm{~B} \mathrm{C}, \mathrm{i}, \mathrm{j}, \mathrm{k})} P(t)) \\
\text { (note that } \mathrm{P}(\mathrm{t}, \mathrm{w}) & =\mathrm{P}(\mathrm{t}))
\end{aligned}
$$

## Computing $\mu$

$$
\mu(A \rightarrow B C, i, j, k)=\sum_{t \text { of this form }} P(t)
$$



## Computing $\mu$



## Computing $\mu$



## Computing $\mu$



## Computing $\mu$

$$
\mu(A \rightarrow B C, i, j, k)=\sum_{t \text { of this form }} P(t)
$$

$$
\mathrm{d}_{1}: \mathrm{S} \Rightarrow^{*} \mathrm{w}_{1} \ldots \mathrm{w}_{\mathrm{i}-1} A \mathrm{w}_{\mathrm{k}} \ldots \mathrm{w}_{\mathrm{n}}
$$

$$
d_{2}: B \Rightarrow^{*} w_{i} \ldots w_{j-1}
$$

$$
\mu(A \rightarrow B C, i, j, k)=\sum_{t \text { of this form }} P(t)
$$

$$
=\sum_{t \text { of this form }} P\left(d_{1}\right) \cdot P(A \rightarrow B C) \cdot P\left(d_{2}\right) \cdot P\left(d_{3}\right)
$$

$$
=\left(\sum_{d_{1}} P\left(d_{1}\right)\right) \cdot P(A \rightarrow B C) \cdot\left(\sum_{d_{2}} P\left(d_{2}\right)\right) \cdot\left(\sum_{d_{3}} P\left(d_{3}\right)\right)
$$

## Computing $\mu$



NB: $\alpha$ and $\beta$ accidentally reversed, compared to literature.

## Inside probabilities

$$
\alpha(A, i, k)=\sum_{t \text { for } B \Rightarrow^{*} w_{i} \ldots w_{k-1}} P(t)
$$



$$
\begin{aligned}
& \alpha(A, i, i+1)=P\left(A \rightarrow w_{i}\right) \\
& \alpha(A, i, k)=\sum_{\substack{A \rightarrow B \\
i<j<k}} P(A \rightarrow B C) \cdot \alpha(B, i, j) \cdot \alpha(C, j, k)
\end{aligned}
$$

## Inside probabilities

$$
\alpha(A, i, k)=\sum_{t \text { for } B \Rightarrow^{*} w_{i} \ldots w_{k-1}} P(t)
$$



## special case:

$P(w)=\alpha(S, 1, n+1)$

$$
\begin{aligned}
& \alpha(A, i, i+1)=P\left(A \rightarrow w_{i}\right) \\
& \alpha(A, i, k)=\sum_{\substack{A \rightarrow B \\
i<j<k}} P(A \rightarrow B C) \cdot \alpha(B, i, j) \cdot \alpha(C, j, k)
\end{aligned}
$$

## Outside probabilities

$$
\beta(A, i, k)=\sum_{t \text { for } S \Rightarrow^{*} w_{1} \ldots w_{i-1} A w_{k} \ldots w_{n}} P(t)
$$

$$
=\sum_{\substack{B \rightarrow A C \\ k<j \leq n}} P(B \rightarrow A C) \cdot \alpha(C, k, j) \cdot \beta(B, i, j)+\sum_{\substack{B \rightarrow C A \\ 1 \leq j<i}} P(B \rightarrow C A) \cdot \alpha(C, j, i) \cdot \beta(B, j, k)
$$



## Outside probabilities

$$
\beta(A, i, k)=\sum_{t \text { for } S \Rightarrow^{*} w_{1} \ldots w_{i-1} A w_{k} \ldots w_{n}} P(t)
$$

$$
=\sum_{\substack{B \rightarrow A C \\ k<j \leq n}} P(B \rightarrow A C) \cdot \alpha(C, k, j) \cdot \beta(B, i, j)+\sum_{\substack{B \rightarrow C \\ 1 \leq j<i}} P(B \rightarrow C A) \cdot \alpha(C, j, i) \cdot \beta(B, j, k)
$$



## The Inside-Outside Algorithm

- Start with some initial estimate of parameters.
- For each sentence $w$, compute $\alpha, \beta$, and $\mu$.
- Compute expected counts $\mathrm{E}(\mathrm{A} \rightarrow \mathrm{B} \mathrm{C})$.
- sum expected counts over all sentences
- remember that $\mathrm{P}(\mathrm{w})=\alpha(\mathrm{S}, 1, \mathrm{n}+1)$
- Re-estimate $\mathrm{P}(\mathrm{A} \rightarrow \mathrm{B} C)$ from expected counts.
- Iterate until convergence.


## Some remarks

- Inside-outside increases likelihood
 in each step.
- But huge problems with local maxima.
- Carroll \& Charniak 92 find 300 different local maxima for 300 different initial parameter estimates.
- Improve by partially bracketing strings (Pereira \& Schabes 92).
- Therefore, EM doesn't really work for totally unsupervised PCFG training.
- But extremely useful in refining existing grammars (Berkeley parser; see next time).


## Summary

- Learning parameters of PCFGs:
- maximum likelihood estimation from raw text
- "hard EM": iterate MLE on Viterbi parses
- EM: use inside-outside algorithm with expected rule counts
- PCFG parsing with MLE parse gets f-score in low 70's. Will improve on this next time (state of the art: 93).
- Have assumed that CFG is given and only parameters are to be learned. Will fix this later in this course.

