n-gram models

Computational Linguistics

Alexander Koller

3 November 2017

Let's play a game

- I will write a sentence on the board.
- Each of you, in turn, gives me a word to continue that sentence, and I will write it down.

Let's play another game

- You write a word on a piece of paper.
- You get to see the piece of paper of your neighbor, but none of the earlier words.
- In the end, I will read the sentence you wrote.

Statistical models in NLP

- *Generative statistical model* of language:
pd P(w) over NL expressions that we can observe.
    - w may be complete sentences or smaller units
    - will later extend this to pd P(w, t) with hidden random variables t

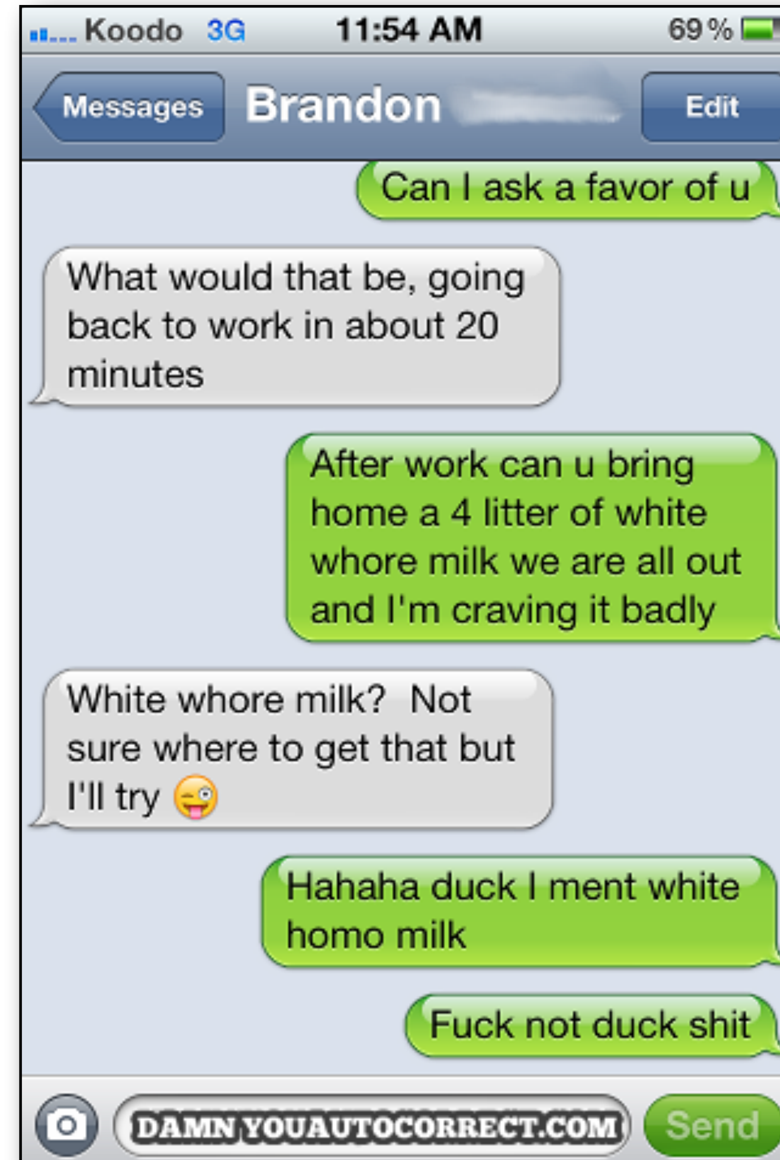
- Assumption: A *corpus* of observed sentences w is generated by repeatedly sampling from P(w).

- We try to estimate the *parameters* of the prob dist from the corpus, so we can make *predictions* about unseen data.

An example


Fido 3G 9:40 PM 56 %
Messages
Anna
Edit
2012-05-17 9:31 PM
Yeah come by
Ok gonna be there in 5 monsoons
So 5 years?
Delivered
DAMN YOUAUTOCORRECT.COM
Send
Q W E R T Y U I O P
A S D F G H J K L
Z X C V B N M
123
space
return


Orange... 12:28 87 %
Messages
Tinkler
Edit
:) . What are your plans for today then?x
Nothing beyond seeing you dead xx
DEAR!!!
DAMN YOUAUTOCORRECT.COM
Send
Q W E R T Y U I O P
A S D F G H J K L
Z X C V B N M
123
space
return


Koodo 3G 11:54 AM 69 %
Messages
Brandon
Edit
Can I ask a favor of u
What would that be, going back to work in about 20 minutes
After work can u bring home a 4 litter of white whore milk we are all out and I'm craving it badly
White whore milk? Not sure where to get that but I'll try
Hahaha duck I ment white homo milk
Fuck not duck shit
DAMN YOUAUTOCORRECT.COM
Send

Word-by-word random process

- A *language model (LM)* is a probability distribution P(w) over sentences.

- Think of it as random process that generates sentences word by word:

X_1	X_2	X_3	X_4
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
copper	is	a	metal

Process from our game

- Each of you = a random variable X_t;
event "$X_t = w_t$" means word at position t is w_t.

- When you chose w_t, you could see the outcomes of the previous variables: $X_1 = w_1, \ldots, X_{t-1} = w_{t-1}$.

- Thus, X_t followed a pd

$$P(X_t = w_t \mid X_1 = w_1, \ldots, X_{t-1} = w_{t-1})$$

Process from our game

- Assume that X_t follows some given PD

$$P(X_t = w_t \mid X_1 = w_1, \ldots, X_{t-1} = w_{t-1})$$

- Then probability of the entire corpus (or sentence) $w = w_1 \ldots w_n$ is joint probability

$$\begin{aligned} P(w_1 \ldots w_n) &= P(w_1) \cdot P(w_2 \mid w_1) \cdot P(w_3 \mid w_1, w_2) \\ &\quad \cdot \ldots \cdot P(w_n \mid w_1, \ldots, w_{n-1}) \end{aligned}$$

How do we estimate these?

Statistical models

- We want to use prob theory to estimate a *model* of a generating process from *observations* about its outcomes.

- Simpler case: we flip a coin 100 times and observe H 61 times. Should we believe that it is a fair coin?
  - observation: absolute freq C(H) = 61, C(T) = 39;
    thus relative freq f(H) = 0.61, f(T) = 0.39
  - model: assume rv X follows a *Bernoulli* distribution,
    i.e. X has two outcomes, and there is a value p such that
    P(X = H) = p and P(X = T) = 1 - p.
  - want to estimate the *parameter* p of this model

Fit of model and observations

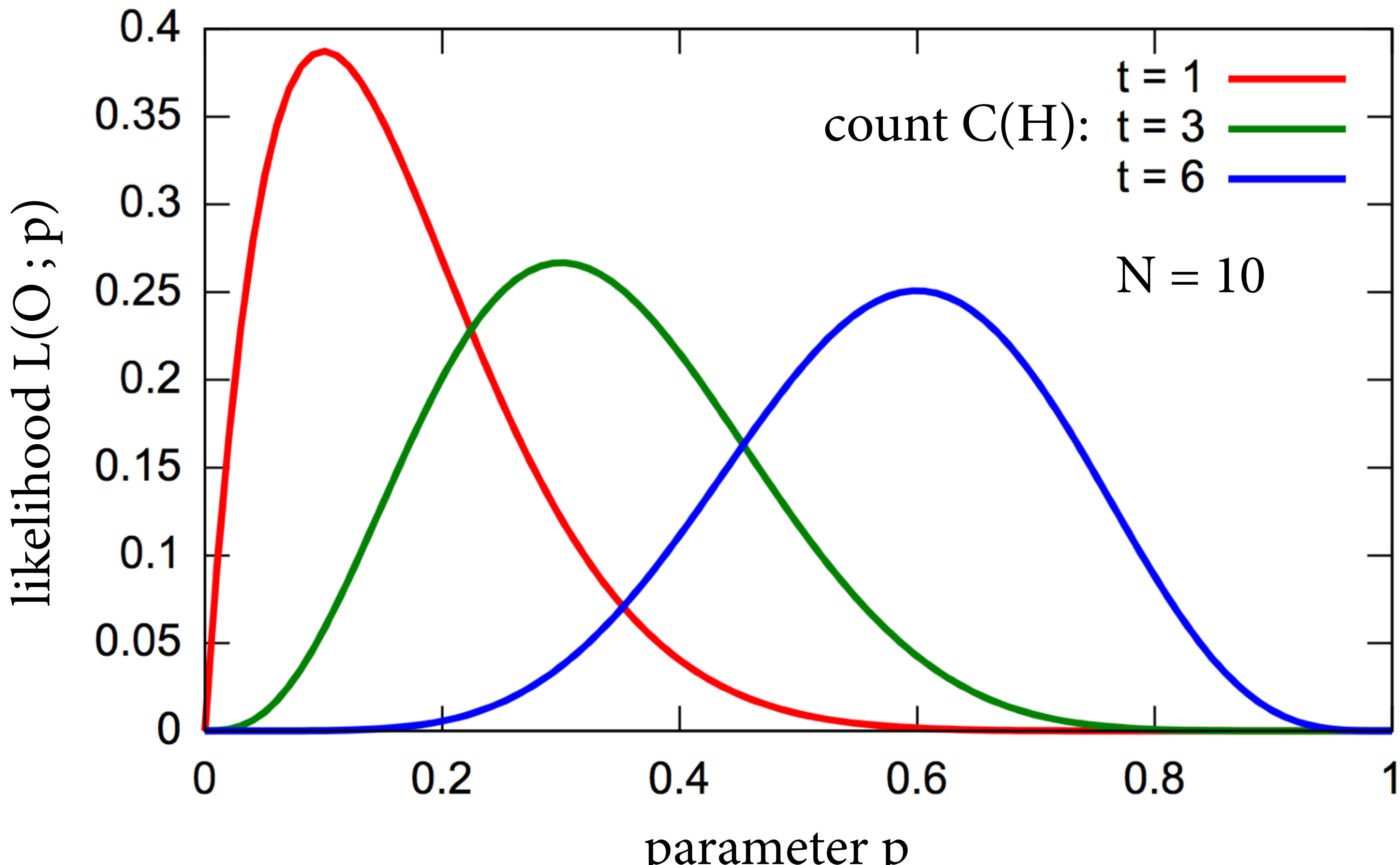
- How do we quantify how well a model fits with the observations we made?

- Out of the many possibilities, easiest is to look at the *likelihood:* probability P(O ; **p**) of the observations O given the values **p** for the model parameters.

- *Maximum likelihood estimation:* find parameter values for which the likelihood of O is maximal.

Likelihood functions

likelihood $L(O\,;\,p) = p^{C(H)} * (1\text{-}p)^{C(T)} * \text{binom}(N, C(H))$



ML Estimation

- Goal: Find value for p that maximizes the likelihood of the observations.

- For Bernoulli models, it is extremely easy to estimate the parameters that maximize the likelihood:
  - P(X = a) = f(a)
  - in the coin example above, just take p = f(H)

- Can prove that relative frequency is an ML estimator for a lot of different statistical models (Bernoulli, multinomial, etc.; see link on course page).

Parameters of the model

- Our model has one parameters for $P(X_t = w_t \mid w_1, \ldots, w_{t-1})$ for all t and $w_1, \ldots, w_t$.

- Can use maximum likelihood estimation:

$$P(w_t \mid w_1, \ldots, w_{t-1}) = \frac{C(w_1 \ldots w_{t-1} w_t)}{C(w_1 \ldots w_{t-1})}$$

- Let's say a natural language has 10^5 different words. How many tuples $w_1, \ldots w_t$ of length t?
  - $t = 1$: 10^5
  - $t = 2$: 10^{10} different contexts
  - $t = 3$: 10^{15}; etc.

Sparse data problem

- Typical corpus sizes:
  - Brown corpus: about 10^6 tokens
  - Gigaword corpus: about 10^9 tokens

- Problem exacerbated by *Zipf's Law:*
  - Order all words by their absolute frequency in corpus (rank 1 = most frequent word).
  - Then log(absolute frequency) falls linearly with log(rank); i.e., most words are really rare.
  - Zipf's Law is very robust across languages and corpora.

Independence assumptions

- Let's pretend that word at position t depends only on the words at positions t-1, t-2, …, t-k for some fixed k (*Markov assumption* of degree k).

- Then we get an n-gram model, with n = k+1:

  $$P(X_t \mid X_1, \ldots, X_{t-1}) = P(X_t \mid X_{t-k}, \ldots, X_{t-1})$$

  for all t.

- Special names for *unigram models* (n = 1), *bigram models* (n = 2), *trigram models* (n = 3).
  - Thus our second game was a bigram model.

Independence assumptions

- We assume statistical independence of X_t from events that are too far in the past, although we know that this assumption is incorrect.

- Typical tradeoff in statistical NLP:
  - if model is too shallow, it won't represent important linguistic dependencies
  - if model is too complex, its parameters can't be estimated accurately from the available data

low *n* ⟷ high *n*

modeling errors ⟷ estimation errors

Bigrams: an example

JOHN READ MOBY DICK MARY READ A DIFFERENT BOOK SHE READ A BOOK BY CHER

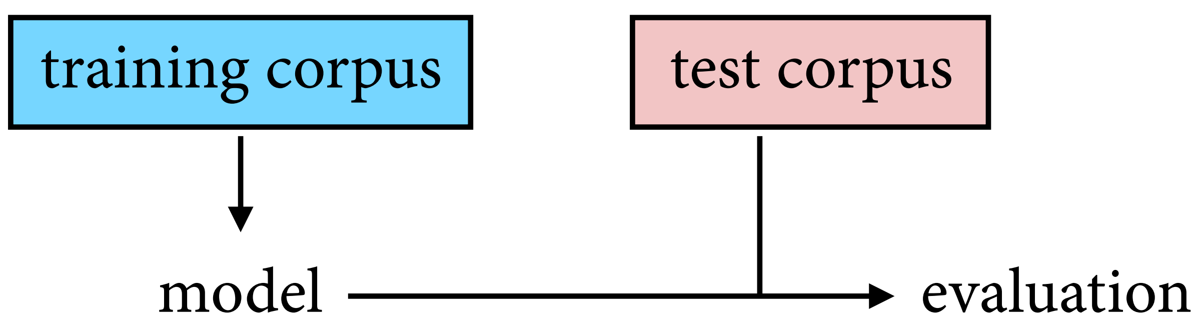
$$
\begin{aligned}
& p(\text{JOHN READ A BOOK}) \\
= \;& p(\text{JOHN}|\bullet) \quad p(\text{READ}|\text{JOHN}) \quad p(\text{A}|\text{READ}) \quad p(\text{BOOK}|\text{A}) \quad p(\bullet|\text{BOOK}) \\
= \;& \frac{c(\bullet\ \text{JOHN})}{\sum_w c(\bullet\ w)} \quad \frac{c(\text{JOHN READ})}{\sum_w c(\text{JOHN}\ w)} \quad \frac{c(\text{READ A})}{\sum_w c(\text{READ}\ w)} \quad \frac{c(\text{A BOOK})}{\sum_w c(\text{A}\ w)} \quad \frac{c(\text{BOOK}\ \bullet)}{\sum_w c(\text{BOOK}\ w)} \\
= \;& \frac{1}{3} \quad \frac{1}{1} \quad \frac{2}{3} \quad \frac{1}{2} \quad \frac{1}{2} \\
\approx \;& 0.06
\end{aligned}
$$

(Chen & Goodman 98)

n-grams: Evaluation

- Measure quality of n-gram model using *perplexity* $PP(w) = P(w_1 \ldots w_N)^{-1/N}$ of test data $w = w_1 \ldots w_N$.

- To get honest picture of model's performance, evaluate it on test data that was not used for training.



- Maximum likelihood model for training corpus is not necessarily good for test corpus (overfitting).

Bigrams: a problem

JOHN READ MOBY DICK
MARY READ A DIFFERENT BOOK
SHE READ A BOOK BY CHER

$$
\begin{aligned}
& p(\text{CHER READ A BOOK}) \\
= \; & p(\text{CHER}|\bullet) \quad p(\text{READ}|\text{CHER}) \quad p(\text{A}|\text{READ}) \quad p(\text{BOOK}|\text{A}) \quad p(\bullet|\text{BOOK}) \\
= \; & \frac{c(\bullet\ \text{CHER})}{\sum_w c(\bullet\ w)} \quad \frac{c(\text{CHER READ})}{\sum_w c(\text{CHER}\ w)} \quad \frac{c(\text{READ A})}{\sum_w c(\text{READ}\ w)} \quad \frac{c(\text{A BOOK})}{\sum_w c(\text{A}\ w)} \quad \frac{c(\text{BOOK}\ \bullet)}{\sum_w c(\text{BOOK}\ w)} \\
= \; & \frac{0}{3} \quad \frac{0}{1} \quad \frac{2}{3} \quad \frac{1}{2} \quad \frac{1}{2} \\
= \; & 0
\end{aligned}
$$

(Chen & Goodman 98)

Unseen data

- ML estimate is "optimal" only for the corpus from which we computed it.
- Usually does not generalize directly to new data.
  - Ok for unigrams, but there are *so many* bigrams.
- ML estimate predicts probability of 0 for n-grams that were not observed in training. This is a disaster because product with 0 is always 0.

Smoothing techniques

- Basic idea: Replace ML estimate

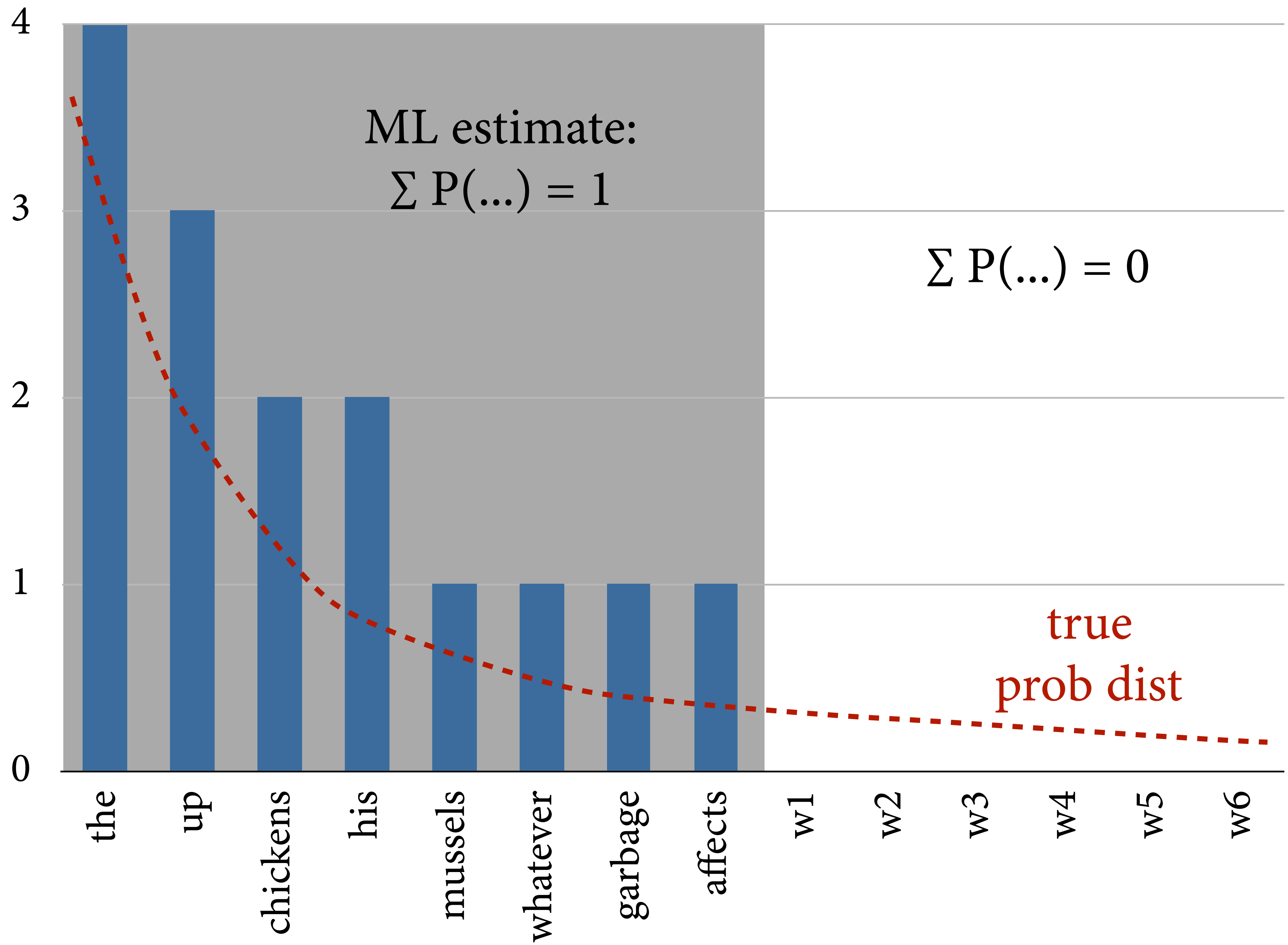
$$P_{\mathrm{ML}}(w_i \mid w_{i-1}) = \frac{C(w_{i-1}w_i)}{C(w_{i-1})}$$

by estimate with adjusted bigram count

$$P^*(w_i \mid w_{i-1}) = \frac{C^*(w_{i-1}w_i)}{C(w_{i-1})}$$

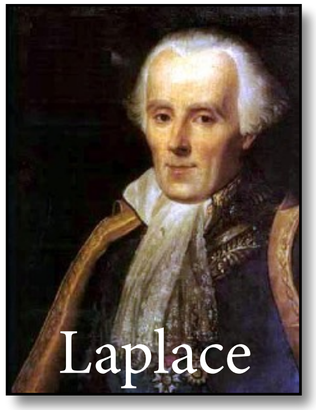
- Redistribute counts from seen to unseen bigrams.
- Generalizes easily to n-gram models with n > 2.

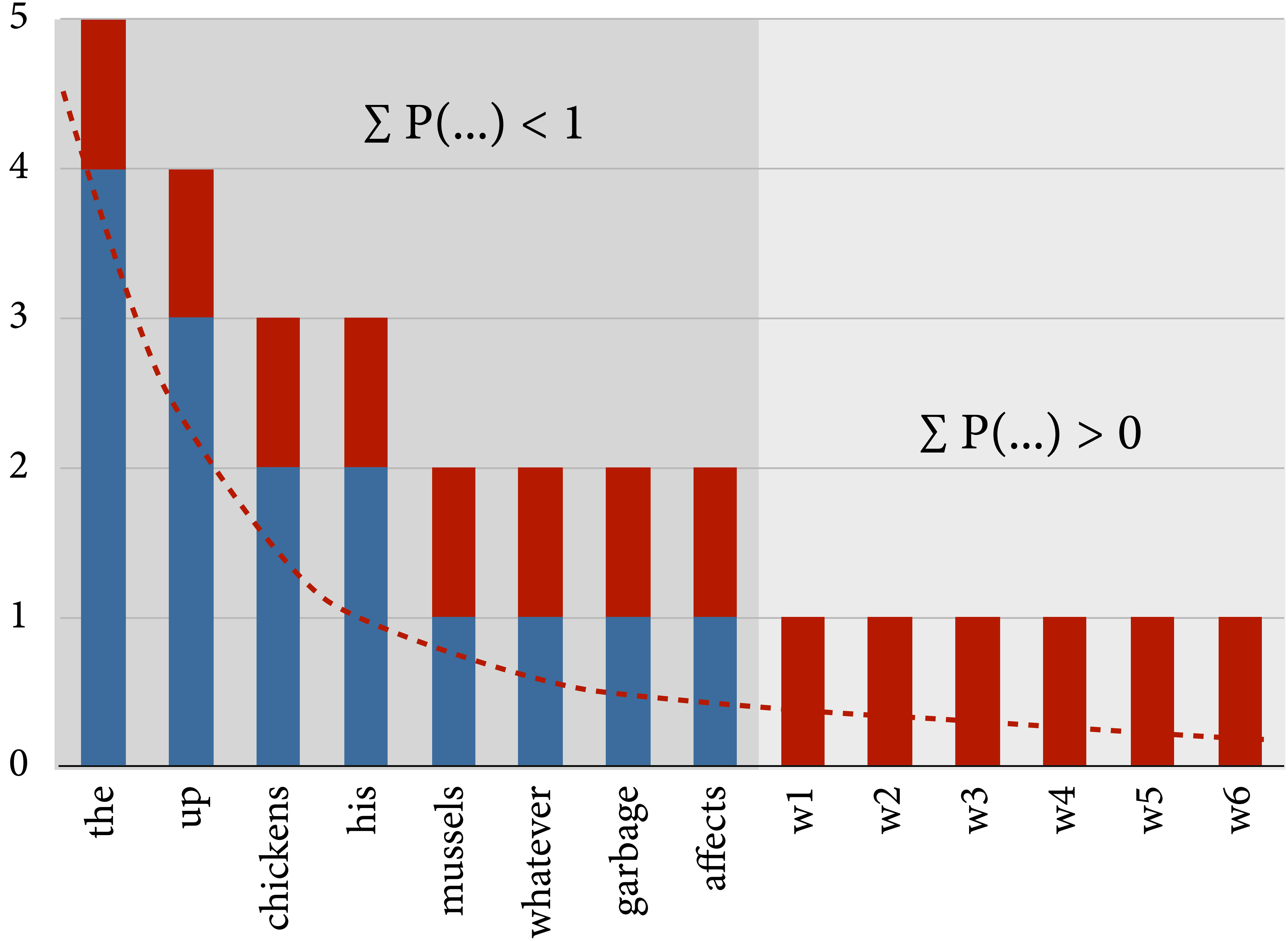
Smoothing



C(eat X) in Brown corpus

Add-one Smoothing





Add-one Smoothing

- Count every bigram (seen or unseen) one more time than in corpus and normalize:

$$P_{\text{lap}}(w_i \mid w_{i-1}) = \frac{C(w_{i-1}w_i)+1}{\sum_w (C(w_{i-1}w)+1)} = \frac{C(w_{i-1}w_i)+1}{C(w_{i-1})+|V|}$$

JOHN READ MOBY DICK
MARY READ A DIFFERENT BOOK
SHE READ A BOOK BY CHER

|V| = 11, |seen bigram types| = 11
⇒ 110 unseen bigrams

$p(\text{JOHN READ A BOOK})$

$$= \frac{1+1}{11+3}\ \frac{1+1}{11+1}\ \frac{1+2}{11+3}\ \frac{1+1}{11+2}\ \frac{1+1}{11+2}$$

$$\approx\ 0.0001$$

$p(\text{CHER READ A BOOK})$

$$= \frac{1+0}{11+3}\ \frac{1+0}{11+1}\ \frac{1+2}{11+3}\ \frac{1+1}{11+2}\ \frac{1+1}{11+2}$$

$$\approx\ 0.00003$$

Add-one Smoothing

- Easy to implement, but dramatically overestimates probability of unseen events.
  - In the Cher example: $P_{lap}(\text{unseen} \mid w_{i-1}) \geq 1/14$; thus "count"($w_{i-1}$ unseen) $\approx 110 * 1/14 = 7.8$.
  - Compare against 12 bigram tokens in training corpus.

- Learn all about how to *really* do smoothing in the course "Statistical NLP".

Conclusion

- Statistical models of natural language.
- Language models with n-grams.
- The problem of data sparseness.
- Smoothing.